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General Physics I, Final 1s
PHYS10000AA, AB, AC, Class year 107
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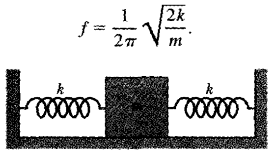
Solutions

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are allowed.

Problems (6 Problems, total 100%)

1. **Spring system:** (15%) Two springs with identical force constant ***k*** are connected as shown in the figure to the right. Prove that the frequency of the oscillation on the frictionless surface is,



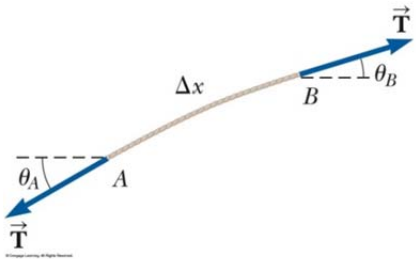
$$f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

2. **Conservation of energy:** (20%) The gravitational force between two particles with masses ***m*** and ***M***, initially at great separation, pulls them together. What is the speed of either particle relative to the other, when ***d*** is the separation at that instant? (10%)

3. **Gravitation:** (15%) The magnitude of the gravitational force between a particle of mass m_1 and one of mass m_2 is given by $F(x) = G\frac{m_1 m_2}{x^2}$, where *G* is a constant, and *x* is the distance between two particles.
   (a) What is the corresponding potential energy function *U (x)*? (5%)
   (b) How much work is required to increase the separation of the particles from $x=x_1$ to $x=x_1+d$? (5%)

4. **Doppler effect:** (15%) If a sound wave has a speed ***v*** and frequency ***f***. What is the detected frequency when the source is moving at speed v_s towards the detector and the detector is stationary? (10%) Derive this.

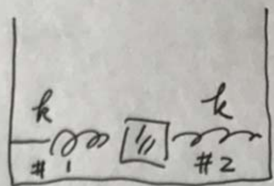
5. **Wave equation**: (20%) Suppose you set up a traveling wave in a string. Refer to the figure to the right, if you focused on this section of the string, you can find the mass of the string is oscillating vertically (y-direction) that is it is perpendicular to the wave's travelling direction (say, to the right or in the +x direction). Let the same section, suppose the vibration of the string can be represented as a function ***y(x, t)***; a function of both ***x*** and ***t***; and let the line density of the string be μ, and ***T*** is the tension of the string. Prove that the wave equation describing this wave motion is



$$\frac{\mu}{T}\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}.$$

6. **Escape Speed of a Rock:** (15%) Superman picks up a 20 Kg rock and throws it into the space. What minimum speed must it have at the Earth's surface to move infinitely far away from the Earth?

1. Consider a displacement of mass m, for example x to the right. The force for mass #1 Spring is $-kx$, for mass by #2 spring is also $-kx$



$$\therefore F_{total} = -kx - kx = -2kx = -k_{eff}\,x$$

$$\therefore k_{eff} = 2k$$

Therefore $f = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

2.

For the two-mass system

$$\frac{1}{2}MV^2 + \frac{1}{2}mv^2 + -\frac{GMm}{d} = 0$$

or $\frac{1}{2}MV^2 + \frac{1}{2}mv^2 = \frac{GMm}{d}$ — ①

From Conservation of linear Momentum $MV = mv = P$

$$\therefore ① \Rightarrow \frac{P^2}{2M} + \frac{P^2}{2m} = \frac{GMm}{d} \quad \Rightarrow P = \sqrt{\frac{2GM^2m^2}{d(M+m)}}$$

Therefore $V_{relative} = v + V = \frac{P}{M} + \frac{P}{m}$

$$= \sqrt{\frac{2G(M+m)}{d}}$$

3

$$U(x) = -\int F(x)\,dx$$

(a) $$U(x) = -\int F(x)\,dx = -\int_x^{\infty} G\frac{m_1 m_2}{x^2}\,dx = -\frac{G m_1 m_2}{x}$$

(b) $$W = U(x_1+d) - U(x_1) = -\frac{G m_1 m_2}{x_1+d} - \left(-\frac{G m_1 m_2}{x_1}\right)$$

$$= \frac{G m_1 m_2 d}{x_1(x_1+d)}$$

4

$S \xrightarrow[V_s]{})))\rightarrow v$ • Detector

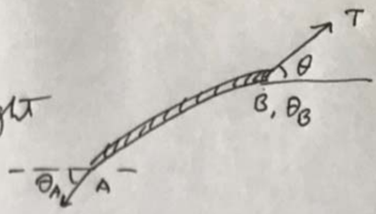
When Source moves towards the detector @ speed V_s, Wave front moves vT during a period of time T,
The Source moves $V_s T$ during the same time period of T, (w
the distance between W_1 and W_2 is the detected λ' for the detector.

$$\lambda' = vT - V_s T$$

$$\therefore f' = \frac{v}{\lambda'} = \frac{v}{vT - V_s T} = \frac{v}{\frac{v}{f} - \frac{V_s}{f}} = f\frac{v}{v - V_s}$$

5

Use the figure to the right



The total force f_y is

$$\sum F_y = T\sin\theta_B - T\sin\theta_A$$

$$= T(\sin\theta_B - \sin\theta_A)$$

$$\doteqdot T(\tan\theta_B - \tan\theta_A)$$

$$= T\left(\frac{\partial y}{\partial x}\Big|_{at\,B} - \frac{\partial y}{\partial x}\Big|_{at\,A}\right) = m a_y = \mu \Delta x\left(\frac{\partial^2 y}{\partial t^2}\right)$$

for small angle $\sin\theta \approx \tan\theta$

Assume line density μ

$$\therefore \mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right) = T\left(\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A\right)$$

or
$$\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A}{\Delta x} = \frac{\partial^2 y}{\partial x^2}$$

Therefore $\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$ or $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v}\frac{\partial^2 y}{\partial t^2}$

is the wave equation describing a wave traveling in the string

6, In an Earth-Object system, the conservation of energy gives

$$\frac{1}{2}mv_i^2 - \frac{GM_E m}{r_i} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f} \quad ; \; r_i = R_E,\; r_f = r_{max}$$

for the rock to escape the earth, $v_f = 0$,

$$\therefore \frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{max}} \quad : \text{To escape, } r_{max} \to \infty$$

$$\therefore \frac{1}{2}mv_{escape}^2 = \frac{GM_E m}{R_E}. \Rightarrow V_{escape} = \sqrt{\frac{2GM_E}{R_E}}$$

roughly $V_{esc} = 1.12\times10^4\ \frac{m}{s}$