Chapter 16.

1. A wave is described by $y = 0.020\,0 \sin(kx - \omega t)$, where $k = 2.11$ rad/m, $\omega = 3.62$ rad/s, x and y are in meters, and t is in seconds. Determine (a) the amplitude, (b) the wavelength, (c) the frequency, and (d) the speed of the wave.

Ans:

Compare the specific equation to the general form:

$$y = (0.0200\,\mathrm{m})\sin(2.11x - 3.62t) = y\,A\sin(kx - \omega t + \phi)$$

(a) $A = \boxed{2.00\text{ cm}}$

(b) $k = 2.11\,\mathrm{rad/m} \rightarrow \lambda = \frac{2\pi}{k} = \boxed{2.98\,\mathrm{m}}$

(c) $\omega = 3.62\,\mathrm{rad/s} \rightarrow f = \frac{\omega}{2\pi} = \boxed{0.576\,\mathrm{Hz}}$

(d) $\upsilon = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{3.62}{2.11} = \boxed{1.72\,\mathrm{m/s}}$

2. The string shown in Figure P16.11 is driven at a frequency of 5.00 Hz. The amplitude of the motion is $A = 12.0$ cm, and the wave speed is $\upsilon = 20.0$ m/s. Furthermore, the wave is such that $y = 0$ at $x = 0$ and $t = 0$. Determine (a) the angular frequency and (b) the wave number for this wave. (c) Write an expression for the wave function. Calculate (d) the maximum transverse speed and (e) the maximum transverse acceleration of an element of the string.

Ans:

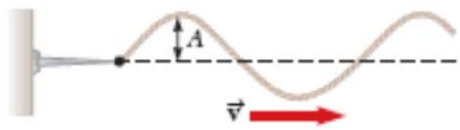

Figure P16.11

(a) $\omega = 2\pi f = 2\pi(5.00\text{ s}^{-1}) = \boxed{31.4\text{ rad/s}}$

(b) $\lambda = \frac{\upsilon}{f} = \frac{20.0\,\mathrm{m/s}}{5.00\,\mathrm{s}^{-1}} = 4.00\,\mathrm{m}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{4.00\,\mathrm{m}} = \boxed{1.57\,\mathrm{rad/m}}$$

(c) In $y = A\sin(kx - \omega t + \phi)$ we take $A = 12.0$ cm. At $x = 0$ and $t = 0$ we have

$y = (12.0\text{ cm})\sin\phi$. To make this fit $y = 0$, we take $\phi = 0$. Then

$\boxed{y = 0.120 \sin(1.57x - 31.4t)\text{, where } x \text{ and } y \text{ are in meters and } t \text{ is in seconds}}$

(d) The transverse velocity is $\frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$.

Its maximum magnitude is

$$A\omega = (12.0\text{ cm})(31.4\text{ rad/s}) = \boxed{3.77\,\mathrm{m/s}}$$

(e) $a_y = \frac{\partial \upsilon_y}{\partial t} = \frac{\partial}{\partial t}\left[-A\omega\cos(kx - \omega t)\right] = -A\omega^2\sin(kx - \omega t)$

The maximum value is $A\omega^2 = (0.120\text{ m})(31.4\text{ s}^{-1})^2 = \boxed{118\text{ m/s}^2}$

3. Transverse pulses travel with a speed of 200 m/s along a taut copper wire whose diameter is 1.50 mm. What is the tension in the wire? (The density of copper is 8.92 g/cm^3.)

Ans:

We use $\upsilon = \sqrt{\frac{T}{\mu}}$ to solve for the tension:

$$T = \mu\upsilon^2 = \rho A\upsilon^2 = p\pi r^2\,\upsilon^2$$
$$T = (8920\ \text{kg/m}^3)(\pi)(7.50 \times 10^{-4}\ \text{m})^2\ (200\ \text{m/s})^2$$
$$T = \boxed{631\ \text{N}}$$

4. A sinusoidal wave on a string is described by the wave function $y = 0.15 \sin (0.80x - 50t)$ where x and y are in meters and t is in seconds. The mass per unit length of this string is 12.0 g/m. Determine (a) the speed of the wave, (b) the wavelength, (c) the frequency, and (d) the power transmitted by the wave.

Ans:

Comparing the given wave function, $y = (0.15) \sin (0.80x - 50t)$, with the general wave function, $y = A \sin (kx - \omega t)$, we have $k = 0.80$ rad/m, $\omega = 50$ rad/s, and $A = 0.15$ m.

(a) $\upsilon = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{\omega}{k} = \frac{50.0}{0.800}\,\text{m/s} = \boxed{62.5\,\text{m/s}}$

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.800}\,\text{m} = \boxed{7.85\,\text{m}}$

(c) $f = \frac{50.0}{2\pi} = \boxed{7.96\,\text{Hz}}$

(d) $P = \frac{1}{2}\mu\omega^2 A^2\upsilon = \frac{1}{2}(12.0\times10^{-3})(50.0)^2(0.150)^2(62.5)\text{W} = \boxed{21.1\text{W}}$

5. Transverse waves travel with a speed of 20.0 m/s on a string under a tension of 6.00 N. What tension is required for a wave speed of 30.0 m/s on the same string?.

Ans:

The two wave speeds can be written as

$$\upsilon_1 = \sqrt{T_1/\mu} \text{ and } \upsilon_2 = \sqrt{T_2/\mu}$$

Since μ is constant, $\mu = \frac{T_2}{\upsilon_2^2} = \frac{T_1}{\upsilon_1^2}$, and

$$T_2 = \left(\frac{\upsilon_2}{\upsilon_1}\right)^2 T_1 = \left(\frac{30.0\,\text{m/s}}{20.0\,\text{m/s}}\right)^2 (6.00\,\text{N}) = \boxed{13.5\,\text{N}}$$