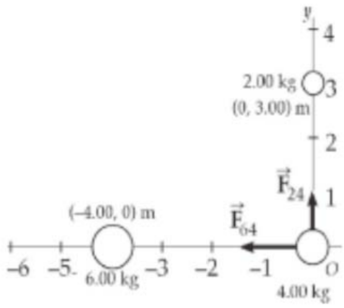
Chapter 13.

1. Three uniform spheres of masses $m_1 = 2.00$ kg, $m_2 = 4.00$ kg, and $m_3 = 6.00$ kg are placed at the corners of a right triangle as shown in Figure P13.6. Calculate the resultant gravitational force on the object of mass m_2, assuming the spheres are isolated from the rest of the Universe.

   Ans:

   The force exerted on the 4.00-kg mass by the 2.00-kg mass is directed upward and given by

$$\begin{aligned}\vec{\mathbf{F}}_{12} &= G\frac{m_2 m_1}{r_{12}^2}\hat{\mathbf{j}} \\ &= (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\\ &\qquad \frac{(4.00\ \text{kg})(2.00\ \text{kg})}{(3.00\ \text{m})^2}\hat{\mathbf{j}} \\ &= 5.93\times10^{-11}\hat{\mathbf{j}}\,\text{N}\end{aligned}$$



ANS. FIG. P13.6

   The force exerted on the 4.00-kg mass by the 6.00-kg mass is directed to the left:

$$\begin{aligned}\vec{\mathbf{F}}_{32} &= G\,\frac{m_2 m_3}{r_{32}^2}(-\hat{\mathbf{i}}) \\ &=(-6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(4.00\ \text{kg})(6.00\ \text{kg})}{(4.00\ \text{m})^2}\hat{\mathbf{i}} \\ &= -10.0\times10^{-11}\hat{\mathbf{i}}\ \text{N}\end{aligned}$$

   Therefore, the resultant force on the 4.00-kg mass is

$$\vec{\mathbf{F}}_4 = \vec{\mathbf{F}}_{24} + \vec{\mathbf{F}}_{64} = \boxed{(-10.0\hat{\mathbf{i}} + 5.93\hat{\mathbf{j}})\times10^{-11}\ \text{N}}$$

2. Three objects of equal mass are located at three corners of a square of edge length ℓ, as shown in Figure P13.15. Find the magnitude and direction of the gravitational field at the fourth corner due to these objects.

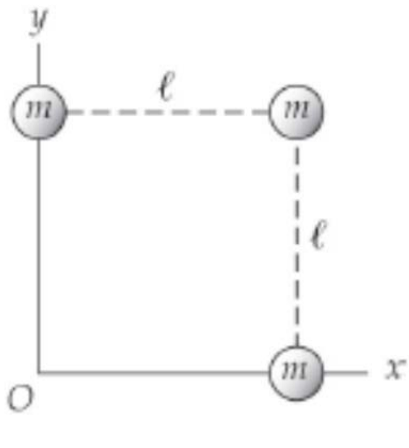
   Ans:

   The vector gravitational field at point O is given by

$$\vec{\mathbf{g}} = \frac{Gm}{l^2}\hat{\mathbf{i}} + \frac{Gm}{l^2}\hat{\mathbf{j}} + \frac{Gm}{2l^2}\left(\cos 45.0°\hat{\mathbf{i}} + \sin 45.0\hat{\mathbf{j}}\right)$$

   So $\vec{\mathbf{g}} = \frac{Gm}{l^2}1 + \left(\frac{1}{2\sqrt{2}}\right)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$

   Or $\boxed{\vec{\mathbf{g}} = \frac{Gm}{l^2}\left(\sqrt{2}\frac{1}{2}\right) \text{toward the opposite corner.}}$



ANS. FIG. P13.15

3. The *Explorer VIII* satellite, placed into orbit November 3, 1960, to investigate the ionosphere, had the following orbit parameters: perigee, 459 km; apogee, 2 289 km (both distances above the Earth's surface); period, 112.7 min. Find the ratio υ_p/υ_a of the speed at perigee to that at apogee.

Ans:

By conservation of angular momentum for the satellite, $r_p\upsilon_p = r_a\upsilon_a$, or

$$\frac{\upsilon_p}{\upsilon_a} = \frac{r_a}{r_p} = \frac{2\ 289\ \text{km} + 6.37\times10^3\ \text{km}}{459\ \text{km} + 6.37\times10^3\ \text{km}} = \frac{8\ 659\ \text{km}}{6\ 829\ \text{km}} = \boxed{1.27}$$

We do not need to know the period.

4. How much work is done by the Moon's gravitational field on a 1 000-kg meteor as it comes in from outer space and impacts on the Moon's surface?

Ans:

The work done by the Moon's gravitational field is equal to the negative of the change of potential energy of the meteor-Moon system:

$$W_{\text{int}} = -\Delta U = -\left(\frac{-Gm_1m_2}{r} - 0?\right)$$

$$W_{\text{int}} = \frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.36\times10^{22}\,\text{kg}\right)\left(1.00\times10^3\ \text{kg}\right)}{1.74\times10^6\ \text{m}}$$

$$= \boxed{2.82\times10^9\ \text{J}}$$

5. A "treetop satellite" moves in a circular orbit just above the surface of a planet, assumed to offer no air resistance. Show that its orbital speed υ and the escape speed from the planet are related by the expression $\upsilon_{\text{esc}} = \sqrt{2}\upsilon$.

Ans:

To obtain the orbital velocity, we use

$$\Sigma F = \frac{mMG}{R^2} = \frac{m\upsilon^2}{R} \quad \text{or} \quad \upsilon = \sqrt{\frac{MG}{R}}$$

We can obtain the escape velocity from

$$\frac{1}{2}m\upsilon_{\text{esc}}^2 = \frac{mMG}{R} \quad \text{or} \quad \upsilon_{\text{esc}} = \sqrt{\frac{2MG}{R}} = \boxed{\sqrt{2}\upsilon}$$

Chapter 14.

1. A large man sits on a four-legged chair with his feet off the floor. The combined mass of the man and chair is 95.0 kg. If the chair legs are circular and have a radius of 0.500 cm at the bottom, what pressure does each leg exert on the floor?

   Ans:

   We shall assume that each chair leg supports one-fourth of the total weight so the normal force each leg exerts on the floor is $n = mg/4$. The pressure of each leg on the floor is then

$$P_{\text{leg}} = \frac{n}{A_{\text{leg}}} = \frac{mg/4}{\pi r^2} = \frac{(95.0\,\text{kg})(9.80\,\text{m/s}^2)}{4\pi\left(0.500\times10^{-2}\,\text{m}\right)^2} = \boxed{2.96\times10^6\ \text{Pa}}$$

2. The spring of the pressure gauge shown in Figure P14.7 has a force constant of 1 250 N/m, and the piston has a diameter of 1.20 cm. As the gauge is lowered into water in a lake, what change in depth causes the piston to move in by 0.750 cm?

   Ans:

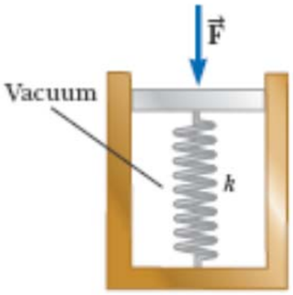


Figure P14.7

Assuming the spring obeys Hooke's law, the increase in force on the piston required to compress the spring an additional amount Δx is

$$\Delta F = F - F_0 = (P - P_0)A = k(\Delta x)$$

The gauge pressure at depth h beneath the surface of a fluid is

$$P - P_0 = \rho g h$$

so we have

$$\rho g h A = k\,(\Delta x)$$

or the required depth is

$$h = k\,(\Delta x)\ \rho g A$$

If $k = 1\ 250$ N/m, $A = \pi d^2/4$, $d = 1.20 \times 10^{-2}$ m, and the fluid is water ($\rho = 1.00 \times 10^3$ kg/m^3), the depth required to compress the spring an additional $\Delta x = 0.750 \times 10^{-2}$ m is $h = 8.46$ m

$$\boxed{h = 8.46\ \text{m}}$$

3. Mercury is poured into a U-tube as shown in Figure P14.22a. The left arm of the tube has cross-sectional area A_1 of 10.0 cm^2, and the right arm has a cross-sectional area A_2 of 5.00 cm^2. One hundred grams of water are then poured into the right arm as shown in Figure P14.22b. (a) Determine the length of the water column in the right arm of the U-tube. (b) Given that the density of mercury is 13.6 g/cm^3, what distance h does the mercury rise in the left arm?

Ans:

(a) Using the definition of density, we have

$$h_\omega = \frac{m_{\text{water}}}{A_2\rho_{\text{water}}}$$

$$= \frac{100\,g}{\left(5.00\,\text{cm}^2\right)\left(1.00\,\text{g/cm}^3\right)}$$

$$= \boxed{20.0\,\text{cm}}$$

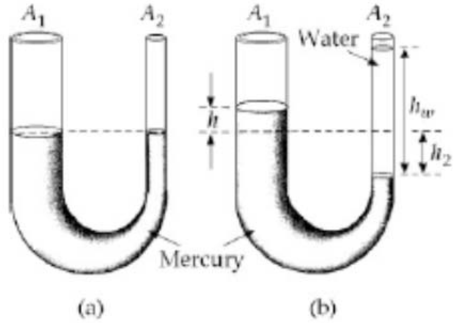


Figure P14.22

(b) represents the situation after the water is added. A volume (A_2h_2) of mercury has been displaced by water in the right tube. The additional volume of mercury now in the left tube is A_1h. Since the total volume of mercury has not changed,

$$A_2h_2 = A_1h \text{ or } h_2 = \frac{A_1}{A_2}h \qquad \textbf{[1]}$$

At the level of the mercury–water interface in the right tube, we may write the absolute pressure as: $P = P_0 + \rho_{\text{water}}gh_\omega$

The pressure at this same level in the left tube is given by

$$P = P_0 + \rho_{\text{Hg}}g(h + h_2) = P_0 + \rho_{\text{water}}gh_\omega$$

which, using equation [1] above, reduces to

$$\rho_{Hg}h\left[1+\frac{A_1}{A_2}\right] = \rho_{\text{water}}h_\omega$$

or $$h = \frac{\rho_{\text{water}}h_\omega}{\rho_{Hg}(1+A_1/A_2)}.$$

Thus, the level of mercury has risen a distance of

$$h = \frac{\left(1.00\,\text{g/cm}^3\right)\left(20.0\,\text{cm}\right)}{\left(13.6\,\text{g/cm}^3\right)\left(1+10.0/5.00\right)}$$

$\boxed{h = 0490\,\text{cm}}$ above the original level.

4. Water moves through a constricted pipe in steady, ideal flow. At the lower point shown in Figure P14.42, the pressure is $P_1 = 1.75 \times 10^4$ Pa and the pipe diameter is 6.00 cm. At another point y = 0.250 m higher, the pressure is $P_2 = 1.20 \times 10^4$ Pa and the pipe diameter is 3.00 cm. Find the speed of flow (a) in the lower section and (b) in the upper section. (c) Find the volume flow rate through the pipe.

Ans:

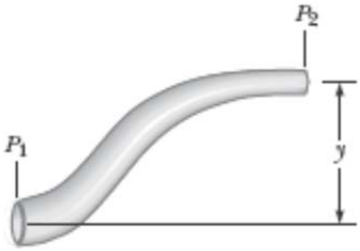


Figure P14.42

(a) The mass flow rate and the volume flow rate are constant:

$$\rho A_1 \upsilon_1 = \rho A_2 \upsilon_2 \rightarrow \pi r_1^2 \upsilon_1 = \pi r_2^2 \upsilon_2$$

Substituting,

$$(3.00 \text{ cm})^2 \upsilon_1 = (1.50 \text{ cm})^2\ \upsilon_2 \rightarrow \upsilon^2 = 4\upsilon_1$$

For ideal flow,

$$P_1 + \rho g y_1 + \frac{1}{2}\rho \upsilon_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho \upsilon_2^2$$

$$1.75\times10^4 \text{ Pa} + 0 + \frac{1}{2}\left(1000 \text{ kg/m}^3\right)\left(\upsilon_1\right)^2$$
$$= 1.20\times10^4 \text{ Pa} + (1000)(9.8)(0.250)\text{ Pa}$$
$$+ \frac{1}{2}\left(1000 \text{ kg/m}^3\right)\left(4\upsilon_1\right)^2$$

Solving for υ_1 gives

$$\upsilon_1 = \sqrt{\frac{3050 \text{ Pa}}{7500 \text{ kg/m}^3}} = \boxed{0.638 \text{ m/s}}$$

(b) From part (a), we have

$$\upsilon_2 = 4\upsilon_1 = \boxed{2.55 \text{m/s}}$$

(c) The volume flow rate is

$$\pi r_1^2 \upsilon_1 = \pi (0.030 \text{ m})^2 (0.638 \text{ m/s}) = \boxed{1.80\times10^{-3} \text{ m}^3/\text{s}}$$

Chapter 15.

1. A 1.00-kg object is attached to a horizontal spring. The spring is initially stretched by 0.100 m, and the object is released from rest there. It proceeds to move without friction. The next time the speed of the object is zero is 0.500 s later. What is the maximum speed of the object?

Ans:

The 0.500 s must elapse between one turning point and the other. Thus the period is 1.00 s.

$$\omega = \frac{2\pi}{T} = 6.28 \text{ s}^{-1} \quad \text{and } \upsilon_{max} = \omega A = (6.28 \text{ s}^{-1})(0.100 \text{ m}) = \boxed{0.628 \text{ m/s}}.$$

2. A block of unknown mass is attached to a spring with a spring constant of 6.50 N/m and undergoes simple harmonic motion with an amplitude of 10.0 cm. When the block is halfway between its equilibrium position and the end point, its speed is measured to be 30.0 cm/s. Calculate (a) the mass of the block, (b) the period of the motion, and (c) the maximum acceleration of the block.

Ans:

(a) Energy is conserved for the block-spring system between the maximum-displacement and the half-maximum points:

$$(K+U)_i = (K+U)_f$$

$$0 + \frac{1}{2}kA^2 = \frac{1}{2}m\upsilon^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2}(6.50\,\mathrm{N/m})(0.100\,\mathrm{m})^2 = \frac{1}{2}m(0.300\,\mathrm{m/s})^2 + \frac{1}{2}(6.50\,\mathrm{N/m})(5.00\times10^{-2}\,\mathrm{m})^2$$

$$3.25\times10^{-2}\,\mathrm{J} = \frac{1}{2}m(0.300\,\mathrm{m/s})^2 + 8.12\times10^{-3}\,\mathrm{J}$$

giving $m = \dfrac{2(2.44\times10^{-2}\,\mathrm{J})}{9.0\times10^{-2}\,\mathrm{m^2/s^2}} = \boxed{0.542\,\mathrm{kg}}$

(b) $$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6.50\,\mathrm{N/m}}{0.542\,kg}} = 3.46\,\mathrm{rad/s}$$

Then, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi\,\mathrm{rad}}{3.46\,\mathrm{rad/s}} = \boxed{1.81\,\mathrm{s}}$

(c) $a_{\max} = A\omega^2 = (0.100\text{ m})(3.46\text{ rad/s})^2 = \boxed{1.20\text{ m/s}^2}$

3. A 326-g object is attached to a spring and executes simple harmonic motion with a period of 0.250 s. If the total energy of the system is 5.83 J, find (a) the maximum speed of the object, (b) the force constant of the spring, and (c) the amplitude of the motion.

Ans:

(a) At the equilibrium position, the total energy of the system is in the form of kinetic energy and $m\upsilon_{\max}^2/2 = E$ so the maximum speed is

$$\upsilon_{\max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(5.83\,\mathrm{J})}{0.326\,kg}} = \boxed{5.98\,\mathrm{m/s}}$$

(b) The period of an object-spring system is $T = 2\pi\sqrt{m/k}$, so the force constant of the spring is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 (0.326\,\text{kg})}{(0.250\,\text{s})^2} = \boxed{206\,\text{N}/m}$$

(c) At the turning points, $x = \pm A$, the total energy of the system is in the form of elastic potential energy, or $E = KA^2/2$, giving the amplitude as

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(5.83\,\text{J})}{206\,\text{N}/m}} = \boxed{0.238\,\text{m}}$$

4. A simple pendulum makes 120 complete oscillations in 3.00 min at a location where $g = 9.80$ m/s^2. Find (a) the period of the pendulum and (b) its length.

Ans:

The period of a pendulum is the time for one complete oscillation and is given by $T = 2\pi\sqrt{\ell/g}$, where ℓ is the length of the pendulum.

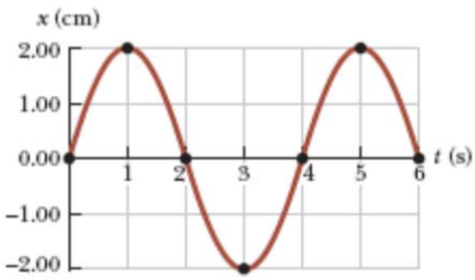
(a) $$T = \left(\frac{3.00\,\text{min}}{120\,\text{oscillations}}\right)\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = \boxed{1.50\text{s}}$$

(b) The length of the pendulum is

$$\ell = g\left(\frac{T^2}{4\pi^2}\right) = \left(9.80\,\text{m/s}^2\right)\left(\frac{(1.50\,\text{s})^2}{4\pi^2}\right) = \boxed{0.559\,\text{m}}$$

5. An object attached to a spring vibrates with simple harmonic motion as described by Figure P15.64. For this motion, find (a) the amplitude, (b) the period, (c) the angular frequency, (d) the maximum speed, (e) the maximum acceleration, and (f) an equation for its position x as a function of time.

Ans:



(a) The amplitude is the magnitude of the maximum displacement from equilibrium (at $x = 0$). Thus, $\boxed{A=2.00\text{cm}}$.

(b) The period is the time for one full cycle of the motion. Therefore, $\boxed{T=4.00\text{ s}}$.

(c) The angular frequency is $\omega = \frac{2\pi}{T} = \frac{2\pi}{4.00\,\text{s}} = \boxed{\frac{\pi}{2}\text{rad/s}}$.

(d) The maximum speed is

$$v_{max} = \omega A = \left(\frac{\pi}{2}\text{ rad/s}\right)(2.00\text{ cm}) = \boxed{\pi\text{ cm/s}}$$

(e) The maximum acceleration is

$$a_{max} = \omega^2 A = \left(\frac{\pi}{2}\text{ rad/s}\right)^2 (2.00\text{ cm}) = \boxed{4.93\text{ cm/s}^2}$$

(f) The general equation for position as a function of time for an object undergoing simple harmonic motion with x = 0 when t = 0 and x increasing positively is $x = A \sin \omega t$. For this oscillator, this becomes

$$\boxed{x = 2.00 \sin\left(\frac{\pi}{2}t\right), \text{ where } x \text{ is in centimeters and } t \text{ in seconds.}}$$