Chapter 12

1. A carpenter's square has the shape of an L as shown in Figure P12.3. Locate its center of gravity.

Ans:

The coordinates of the center of gravity of piece 1 are

$$x_1 = 2.00 \text{ cm and } y_1 = 9.00 \text{ cm}$$

The coordinates for piece 2 are

$$x_2 = 8.00 \text{ cm and } y_2 = 2.00 \text{ cm}$$

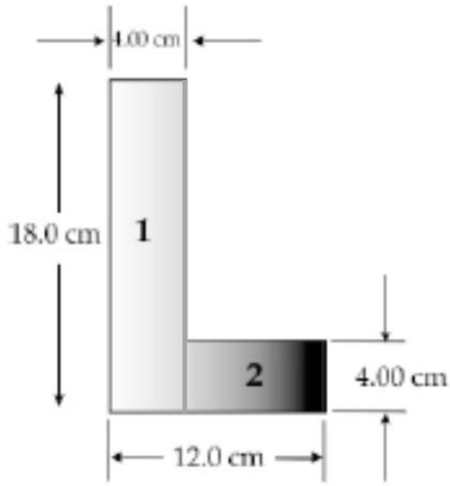
The area of each piece is

$$A_1 = 72.0 \text{ cm}^2 \text{ and } A_2 = 32.0 \text{ cm}^2$$

And the mass of each piece is proportional to the area. Thus,

$$x_{\mathrm{CG}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(72.0\,\mathrm{cm}^2)(2.00\,\mathrm{cm}) + (32.0\,\mathrm{cm}^2)(8.00\,\mathrm{cm})}{72.0\,\mathrm{cm}^2 + 32.0\,\mathrm{cm}^2}$$

$$= \boxed{3.85\,\mathrm{cm}}$$



ANS. FIG. P12.3

and

$$y_{\mathrm{CG}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(72.0\,\mathrm{cm}^2)(9.00\,\mathrm{cm}) + (32.0\,\mathrm{cm}^2)(2.00\,\mathrm{cm})}{104\,\mathrm{cm}^2}$$

$$= \boxed{6.85\,\mathrm{cm}}$$

2. Find the mass m of the counterweight needed to balance a truck with mass M = 1 500 kg on an incline of θ = 45º (Fig. P12.9). Assume both pulleys are friction-less and massless.

Ans:

The second condition for equilibrium at the pulley is

$$\Sigma\tau = 0 = mg(3r) - Tr$$

and from equilibrium at the truck, we obtain

$$2T - Mg\sin 45.0^\circ = 0$$

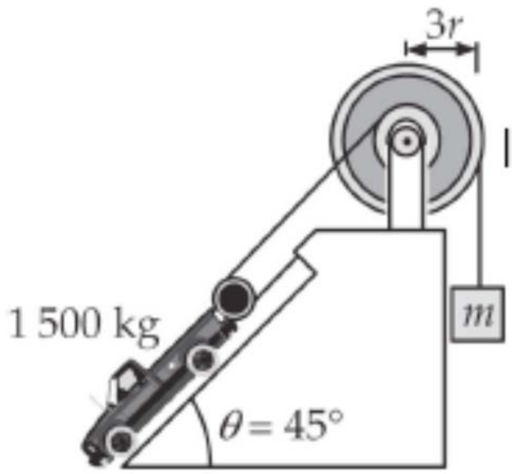
$$T = \frac{Mg\sin 45.0^\circ}{2}$$

$$= \frac{(1500\,\text{kg})g\sin 45.0^\circ}{2}$$

$$= 530g\ \text{N}$$

solving for the mass of the counterweight from [1] and substituting gives

$$m = \frac{T}{3g} = \frac{530g}{3g} = \boxed{177\,\text{kg}}$$



ANS. FIG. P12.9

3. A uniform beam of length 7.60 m and weight 4.50×10^2 N is carried by two workers, Sam and Joe, as shown in Figure P12.11. Determine the force that each person exerts on the beam.

Ans:

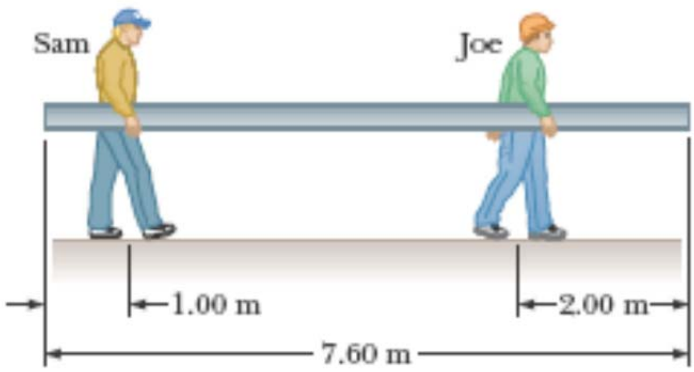


Figure P12.11

Since the beam is in equilibrium, we choose the center as our pivot point and require that

$$\Sigma\, \tau_{\text{center}} = -F_{\text{Sam}}\,(2.80\text{ m}) + F_{\text{Joe}}\,(1.80\text{ m}) = 0$$

or

$$F_{\text{Joe}} = 1.56 F_{\text{Sam}} \qquad \textbf{[1]}$$

Also,

$$\Sigma F_y = 0 \Rightarrow F_{\text{Sam}} + F_{\text{Joe}} = 450\text{ N} \qquad \textbf{[2]}$$

Substitute equation [1] into [2] to get the following:

$$F_{\text{Sam}} + 1.56 F_{\text{Sam}} = 450\text{ N or } F_{\text{Sam}} = \frac{450\text{ N}}{2.56} = 176\text{ N}$$

Then, equation [1] yields $F_{\text{Joe}} = 1.56(176\text{ N}) = 274\text{ N}$

$\boxed{\text{Sam exerts an upward force of 176 N.}}$

$\boxed{\text{Joe exerts an upward force of 274 N.}}$

4. A steel wire of diameter 1 mm can support a tension of 0.2 kN. A steel cable to support a tension of 20 kN should have diameter of what order of magnitude?

Ans:

Count the wires. If they are wrapped together so that all support nearly equal stress, the number should be

$$\frac{20.0\,\mathrm{kN}}{0.200\,\mathrm{kN}}=100$$

Since cross-sectional area is proportional to diameter squared, the diameter of the cable will be

$$(1\,\mathrm{mm})\sqrt{100}\boxed{\sim 1\,\mathrm{cm}}$$

5. A 1 200-N uniform boom at ϕ = 65° to the vertical is supported by a cable at an angle θ = 25.0° to the horizontal as shown in Figure P12.46. The boom is pivoted at the bottom, and an object of weight m = 2 000 N hangs from its top. Find (a) the tension in the support cable and (b) the components of the reaction force exerted by the floor on the boom.

Ans:

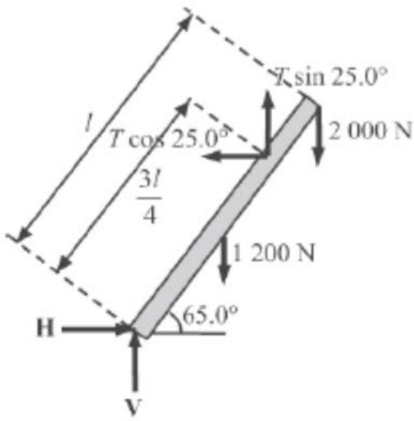
ANS. FIG. P12.46 shows the force diagram.

$\Sigma\tau_{\text{point O}} = 0$ gives

$$(T\cos 25.0^\circ)\left(\frac{3\ell}{4}\sin 65.0^\circ\right)+(T\sin 25.0^\circ)\left(\frac{3\ell}{4}\cos 65.0^\circ\right)$$

$$=(2\,000\,\mathrm{N})(\ell\cos 65.0^\circ)+(1\,200\,\mathrm{N})\left(\frac{\ell}{2}\cos 65.0^\circ\right)$$

From which, T = 1 465 N = $\boxed{1.46\text{ kN}}$



ANS. FIG. P12.46

From $\Sigma F_x = 0$,

$H = T\cos 25.0°$ = 1 328 N(toward right) = $\boxed{1.33\text{ kN}}$

From $\Sigma F_y = 0$,

V = 3 200 N − $T\sin 25.0°$ = 2 581 N(upward) = $\boxed{2.58\text{ kN}}$