Chapter 10

1. (a) Find the angular speed of the Earth's rotation about its axis. (b) How does this rotation affect the shape of the Earth?

Ans:

(a) The Earth rotates 2π radians (360°) on its axis in 1 day. Thus,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{1 \cancel{\text{day}}}\left(\frac{1 \cancel{\text{day}}}{8.64 \times 10^4 \text{ s}}\right) = \boxed{7.27 \times 10^{-5} \text{ rad/s}}$$

(b) Because of its angular speed, $\boxed{\text{the Earth bulges at the equator}}$.

2. A dentist's drill starts from rest. After 3.20 s of constant angular acceleration, it turns at a rate of 2.51×10^4 rev/min. (a) Find the drill's angular acceleration. (b) Determine the angle (in radians) through which the drill rotates during this period.

Ans:

We are given $\omega_f = 2.51 \times 10^4$ rev/min $= 2.63 \times 10^3$ rad/s

(a) $\alpha = \dfrac{\omega_f - \omega_i}{t} = \dfrac{2.63 \times 10^3 \text{ rad/s} - 0}{3.20 \text{ s}} = 8.21 \times 10^2 \text{ rad/s}^2$

(b) $\theta_f = \omega_i t + \dfrac{1}{2}\alpha t^2 = 0 + \dfrac{1}{2}(8.21 \times 10^2 \text{ rad/s}^2)(3.20 \text{ s})^2 = \boxed{4.21 \times 10^3 \text{ rad}}$

3. A car accelerates uniformly from rest and reaches a speed of 22.0 m/s in 9.00 s. Assuming the diameter of a tire is 58.0 cm, (a) find the number of revolutions the tire makes during this motion, assuming that no slipping occurs. (b) What is the final angular speed of a tire in revolutions per second?

Ans:

(a) We first determine the distance travelled by the car during the 9.00-s interval:

$$s = \bar{\upsilon}t = \frac{\upsilon_i + \upsilon_f}{2}t = (11.0 \text{ m/s})(9.00 \text{ s}) = 99.0 \text{ m}$$

the number of revolutions completed by the tire is then

$$\theta = \frac{s}{r} = \frac{99.0 \text{ m}}{0.290 \text{ m}} = 341 \text{ rad} = \boxed{54.3 \text{ rev}}$$

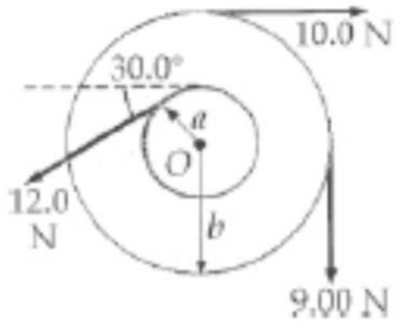
(b) $\omega_f = \frac{\upsilon_f}{r} = \frac{22.0 \text{ m/s}}{0.290 \text{ m}} = 75.9 \text{ rad/s} = \boxed{12.1 \text{ rev/s}}$

4. Find the net torque on the wheel in Figure P10.27 about the axle through *O*, taking *a* = 10.0 cm and *b* = 25.0 cm.

Ans:

To find the net torque, we add the individual torques, remembering to apply the convention that a torque producing clockwise rotation is negative and a counterclockwise rotation is positive.

$$\begin{aligned}\sum \tau &= (0.100 \text{ m})(120 \text{ N}) \\ &\quad - (0.250 \text{ m})(9.00 \text{ N}) \\ &\quad - (0.250 \text{ m})(10.0 \text{ N}) \\ &= \boxed{-3.35 \text{ N} \cdot \text{m}}\end{aligned}$$



The thirty-degree angle is unnecessary information.

ANS. FIG. P10.27

5. A 150-kg merry-go-round in the shape of a uniform, solid, horizontal disk of radius 1.50 m is set in motion by wrapping a rope about the rim of the disk and pulling on the rope. What constant force must be exerted on the rope to bring the merry-go-round from rest to an angular speed of 0.500 rev/s in 2.00 s?

Ans:

We first determine the moment of inertia of the merry-go-round:

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(150\text{ kg})(1.50\text{ m})^2 = 169\text{ kg}\cdot\text{m}^2$$

To find the angular acceleration, we use

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \left(\frac{0.500\text{ rev/s} - 0}{2.00\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \frac{\pi}{2}\text{ rad/s}^2$$

From the definition of torque, $\tau = F{\cdot}r = I\alpha$ we obtain

$$F = \frac{I\alpha}{r} = \frac{(169\text{ kg}\cdot\text{m}^2)\left(\frac{\pi}{2}\text{ rad/s}^2\right)}{1.50\text{ m}} = \boxed{177\text{ N}}$$

Chapter 11

1. The displacement vectors 42.0 cm at 15.0° and 23.0 cm at 65.0° both start from the origin and form two sides of a parallelogram. Both angles are measured counterclockwise from the x axis. (a) Find the area of the parallelogram. (b) Find the length of its longer diagonal.

Ans:

(a)

$$\text{area} = \left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = AB\sin\theta = (42.0\text{ cm})(23.0\text{ cm})\sin(65.0°-15.0°)$$
$$= \boxed{740\text{ cm}^2}$$

(b) The longer diagonal is equal to the sum of the two vectors.

$$\vec{\mathbf{A}}+\vec{\mathbf{B}} = \left[(42.0\text{ cm})\cos 15.0° + (23.0\text{ cm})\cos 65.0°\right]\hat{\mathbf{i}}$$
$$+\left[(42.0\text{ cm})\right]\sin 15.0° + (23.0\text{ cm})\sin 65.0°]\hat{\mathbf{j}}$$

$$\vec{\mathbf{A}}+\vec{\mathbf{B}} = (50.3\text{ cm})\hat{\mathbf{i}} + (31.7\text{ cm})\hat{\mathbf{j}}$$
$$\text{length} = \left|\vec{\mathbf{A}}+\vec{\mathbf{B}}\right| = \sqrt{(50.3\text{ cm})^2 + (31.7\text{ cm})^2} = \boxed{59.5\text{ cm}}$$

2. The position vector of a particle of mass 2.00 kg as a function of time is given by $\vec{\mathbf{r}} = \left(6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}}\right)$, where $\vec{\mathbf{r}}$ is in meters and t is in seconds. Determine the angular momentum of the particle about the origin as a function of time.

Ans:

Differentiating $\vec{\mathbf{r}} = (6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}}\text{m})$ with respect to time gives

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = 5.00\hat{\mathbf{j}}\text{m/s}$$

so $\vec{\mathbf{p}} = m\vec{\mathbf{v}} = (2.00\text{ kg})\left(5.00\hat{\mathbf{j}}\text{m/s}\right) = 10.0\hat{\mathbf{j}}\text{kg}\cdot\text{m/s}$

and $$\vec{\mathbf{L}} = \vec{\mathbf{r}}\times\vec{\mathbf{p}} = \begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 6.00 & 5.00t & 0\\ 0 & 10.0 & 0\end{vmatrix} = \left(60.0\text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}$$

3. A uniform solid disk of mass $m = 3.00$ kg and radius $r = 0.200$ m rotates about a fixed axis perpendicular to its face with angular frequency 6.00 rad/s. Calculate the magnitude of the angular momentum of the disk when the axis of rotation (a) passes through its center of mass and (b) passes through a point midway between the center and the rim.

Ans:

(a) For an axis of rotation passing through the center of mass, the magnitude of the angular momentum is given by

$$L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}(3.00\ \text{kg})(0.200\ \text{m})^2(6.00\ \text{rad/s})$$

$$= \boxed{0.360\ \text{kg}\cdot\text{m}^2/\text{s}}$$

(b) For a point midway between the center and the rim, we use the parallel-axis theorem to find the moment of inertia about this point. Then,

$$L = I\omega = \left[\frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2\right]\omega$$

$$= \frac{3}{4}(3.00\ \text{kg})(0.200\ \text{m})^2(6.00\ \text{rad/s}) = \boxed{0.540\,\text{kg}\cdot\text{m}^2/\text{s}}$$

4. A playground merry-go-round of radius $R = 2.00$ m has a moment of inertia $I = 250\ \text{kg}\cdot\text{m}^2$ and is rotating at 10.0 rev/min about a frictionless, vertical axle. Facing the axle, a 25.0-kg child hops onto the merry-go-round and manages to sit down on the edge. What is the new angular speed of the merry-go-round?

Ans:

From conservation of angular momentum,

$$I_i\,\omega_i = I_f\omega_f\text{: } (250\ \text{kg}\cdot\text{m}^2)(10.0\ \text{rev/min}) = [250\ \text{kg}\cdot\text{m}^2 + (25.0\ \text{kg})(2.00\ \text{m})^2]\omega_2$$

$$\omega_2 = \boxed{7.14\ \text{rev/min}}$$