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General Physics- II, Quiz 9s
PHYS10000AA, Fall Semester-107
2019-06-04

St. ID:__________________________, Name:__________________________

Note: You can use pencil or any pen in answering the problems. Dictionary, calculators and mathematics tables are allowed. Please hand in both solution and this problem sheet.
ABSOLUTELY NO CHEATING!

Problems (total 4 problems, 100%)

1. A circular loop of wire of radius 12.0 cm is placed in a magnetic field directed perpendicular to the plane of the loop as in Figure P30.1. If the field decreases at the rate of 0.050 0 T/s in some time interval, find the magnitude of the emf induced in the loop during this interval. (25%)
Ans: 2.26 mV

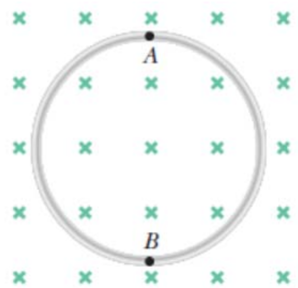


Figure P30.1

With the field directed perpendicular to the plane of the coil, the flux through the coil is $\Phi_B = BA\cos 0^o = BA$. As the magnitude of the field increases, the magnitude of the induced emf in the coil is

$$|\mathcal{E}| = \frac{|\Delta\Phi_B|}{\Delta t} = \left(\frac{\Delta B}{\Delta t}\right)A = (0.050\,0\ \text{T/s})\left[\pi(0.120\ \text{m})^2\right]$$
$$= 2.26\times10^{-3}\ \text{V} = \boxed{2.26\ \text{mV}}$$

2. A long solenoid has n = 400 turns per meter and carries a current given by $I = 30.0(1-e^{-1.60t})$, where I is in amperes and t is in seconds. Inside the solenoid and coaxial with it is a coil that has a radius of R = 6.00 cm and consists of a total of N = 250 turns of fine wire (Fig. P30.4). What emf is induced in the coil by the changing current? (25%)
Ans: $\varepsilon = 68.2e^{-1.60t}$

The solenoid creates a magnetic field

$$B = \mu_0 nI = (4\pi\times10^{-7}\ \text{N/A}^2)(400\ \text{turns/m})(30.0\ \text{A})(1-e^{-1.60\,t})$$

$$B = (1.51\ \times\ 10^{-2}\ \text{N/m}\cdot\text{A})(1-e^{-1.60\,t})$$

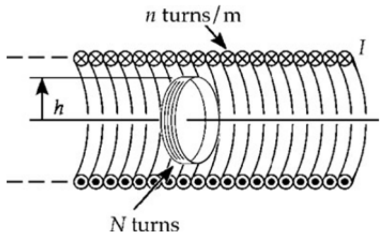
The magnetic flux through one turn of the flat coil is $\Phi_B = \int B\,dA\cos\theta$, but since $dA\cos\theta$ refers to the area perpendicular to the flux, and the magnetic field is uniform over the area A of the flat coil, this integral simplifies to

$$\Phi_B = B\int dA = B\left(\pi R^2\right)$$
$$= \left(1.51\times10^{-2}\ \text{N/m}\cdot\text{A}\right)\left(1-e^{-1.60t}\right)\left[\pi(0.060\ 0\ \text{m})^2\right]$$
$$= \left(1.71\times10^{-4}\ \text{N/m}\cdot\text{A}\right)\left(1-e^{-1.60t}\right)$$

The emf generated in the N-turn coil is $\mathcal{E} = -N\, d\Phi_B/dt$. Because t has the standard unit of seconds, the factor 1.60 must have the unit s^{-1}.

$$\mathcal{E} = -(250)\left(1.71\times10^{-4}\frac{\text{N}\cdot\text{m}}{\text{A}}\right)\frac{d\left(1-e^{-1.60\,t}\right)}{dt}$$
$$= -\left(0.042\ 6\frac{\text{N}\cdot\text{m}}{\text{A}}\right)(1.60\ \text{s}^{-1})e^{t-1.60}$$

$\boxed{\mathcal{E} = 68.2e^{-1.60t}\text{, where } t \text{ is in seconds and } \mathcal{E} \text{ is in mV.}}$



ANS. FIG. P30.4

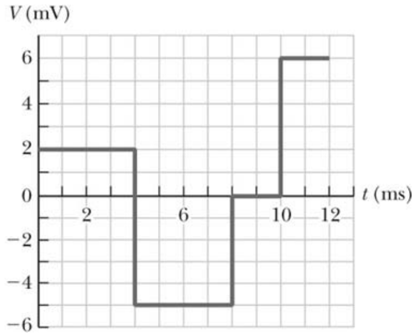
3. A 10.0-mH inductor carries a current $i = I_{max}\sin vt$, with $I_{max} = 5.00$ A and $f = \omega/2\pi = 60.0$ Hz. What is the self-induced emf as a function of time? (25%)
Ans: ε = -18.8cos120πt

Using the definition of self-inductance, $\mathcal{E} = -L\dfrac{di}{dt}$, we obtain

$$\mathcal{E} = -L\frac{d}{dt}\left(I_i\sin\omega t\right) = -L\omega\left(I_i\cos\omega t\right)$$
$$= -\left(10.0\times10^{-3}\right)\left[2\pi(60.0)\right](5.00)\cos\omega t$$

$\boxed{\mathcal{E} = -18.8\cos 120\pi t\text{, where } \mathcal{E} \text{ is in volts and } t \text{ is in seconds.}}$

4. The current in a 4.00 mH-inductor varies in time as shown in Figure P31.8. Construct a graph of the self-induced emf across the inductor over the time interval $t = 0$ to $t = 12.0$ ms. (25%)

Ans: 

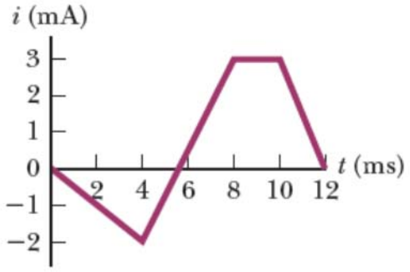


Figure P31.8

The current change is linear, so

$$\mathcal{E} = -L\frac{di}{dt} = -L\frac{\Delta i}{\Delta t}.$$

$t = 0$ to 4 ms:

$$\mathcal{E} = -(4.00\text{ mH})\frac{-2.00\text{ mA}}{4.00\text{ ms}} = +2.00\text{ mV}$$

$t = 4$ to 8 ms:

$$\mathcal{E} = -(4.00\text{ mH})\frac{+5.00\text{ mA}}{4.00\text{ ms}} = -5.00\text{ mV}$$

$t = 8$ to 10 ms:

$$\mathcal{E} = -(4.00\text{ mH})\frac{0}{2.00\text{ ms}} = 0.00\text{ mV}$$

$t = 10$ to 12 ms:

$$\mathcal{E} = -(4.00\text{ mH})\frac{-3.00\text{ mA}}{2.00\text{ ms}} = +6.00\text{ mV}$$