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General Physics-I, Quiz 4
PHYS10000AA, Fall Semester-107
2018-12-18

St. ID:________________________, Name:________________________

Note: You can use pencil or any pen in answering the problems. Dictionary, calculators and mathematics tables are allowed. Please hand in both solution and this problem sheet.
ABSOLUTELY NO CHEATING!

Problems (total 4 problems, 120%)

1. A 326-g object is attached to a spring and executes simple harmonic motion with a period of 0.250 s. If the total energy of the system is 5.83 J, find (a) the maximum speed of the object (10%), (b) the force constant of the spring (10%), and (c) the amplitude of the motion. (10%)

Ans:

(a) At the equilibrium position, the total energy of the system is in the form of kinetic energy and $m\upsilon_{\max}^2/2 = E$ so the maximum speed is

$$\upsilon_{\max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(5.83\,\mathrm{J})}{0.326\,kg}} = \boxed{5.98\,\mathrm{m/s}}$$

(b) The period of an object-spring system is $T = 2\pi\sqrt{m/k}$, so the force constant of the spring is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.326\,\mathrm{kg})}{(0.250\,\mathrm{s})^2} = \boxed{206\,\mathrm{N}/m}$$

(c) At the turning points, $x = \pm A$, the total energy of the system is in the form of elastic potential energy, or $E = KA^2/2$, giving the amplitude as

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(5.83\,\mathrm{J})}{206\,\mathrm{N}/m}} = \boxed{0.238\,\mathrm{m}}$$

2. An object attached to a spring vibrates with simple harmonic motion as described by Figure P15.64. For this motion, find (a) the amplitude (5%), (b) the period (5%), (c) the angular frequency (5%), (d) the maximum speed (5%), (e) the maximum acceleration (5%), and (f) an equation for its position x as a function of time. (5%)

Ans:

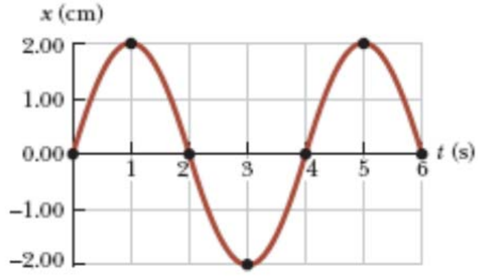

Figure P15.64

(a) The amplitude is the magnitude of the maximum displacement from equilibrium (at x = 0). Thus, $\boxed{A\text{=2.00cm}}$.

(b) The period is the time for one full cycle of the motion. Therefore, $\boxed{\text{T=4.00 s}}$.

(c) The angular frequency is $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{4.00\,\text{s}} = \boxed{\dfrac{\pi}{2}\,\text{rad/s}}$.

(d) The maximum speed is

$$\upsilon_{\max} = \omega A = \left(\frac{\pi}{2}\,\text{rad/s}\right)(2.00\,\text{cm}) = \boxed{\pi\,\text{cm/s}}$$

(e) The maximum acceleration is

$$a_{\max} = \omega^2 A = \left(\frac{\pi}{2}\,\text{rad/s}\right)^2 (2.00\,\text{cm}) = \boxed{4.93\,\text{cm/s}^2}$$

(f) The general equation for position as a function of time for an object undergoing simple harmonic motion with x = 0 when t = 0 and x increasing positively is x=A sin ωt. For this oscillator, this becomes

$$\boxed{x = 2.00\sin\left(\frac{\pi}{2}t\right), \text{where } x \text{ is in centimeters and } t \text{ } in \text{ seconds.}}$$

3. The string shown in Figure P16.11 is driven at a frequency of 5.00 Hz. The amplitude of the motion is A = 12.0 cm, and the wave speed is υ = 20.0 m/s. Furthermore, the wave is such that y = 0 at x = 0 and t = 0. Determine (a) the angular frequency (6%) and (b) the wave number for this wave (6%). (c) Write an expression for the wave function (6%). Calculate (d) the maximum transverse speed (6%) and (e) the maximum transverse acceleration of an element of the string. (6%)

Ans:

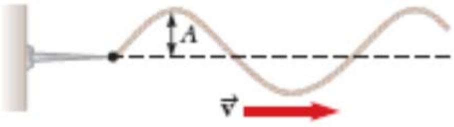

Figure P16.11

(a) $\omega = 2\pi f = 2\pi(5.00\ \mathrm{s}^{-1}) = \boxed{31.4\ \mathrm{rad/s}}$

(b) $\lambda = \dfrac{\upsilon}{f} = \dfrac{20.0\,\mathrm{m/s}}{5.00\,\mathrm{s}^{-1}} = 4.00\,\mathrm{m}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{4.00\,\mathrm{m}} = \boxed{1.57\,\mathrm{rad/m}}$$

(c) In $y = A\sin(kx - \omega t + \phi)$ we take A = 12.0 cm. At x = 0 and t = 0 we have

$y = (12.0\ \mathrm{cm})\sin\phi$. To make this fit y = 0, we take $\phi = 0$. Then

$\boxed{y = 0.120 \sin(1.57x - 31.4t),\ \text{where } x \text{ and } y \text{ are in meters and } t \text{ is in seconds}}$

(d) The transverse velocity is $\dfrac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$.

Its maximum magnitude is

$$A\omega = (12.0\ \mathrm{cm})(31.4\ \mathrm{rad/s}) = \boxed{3.77\,\mathrm{m/s}}$$

(e) $a_y = \dfrac{\partial \upsilon_y}{\partial t} = \dfrac{\partial}{\partial t}\left[-A\omega\cos(kx - \omega t)\right] = -A\omega^2\sin(kx - \omega t)$

The maximum value is $A\omega^2 = (0.120\ \mathrm{m})(31.4\ \mathrm{s}^{-1})^2 = \boxed{118\ \mathrm{m/s}^2}$

4. Transverse waves travel with a speed of 20.0 m/s on a string under a tension of 6.00 N. What tension is required for a wave speed of 30.0 m/s on the same string? (30%)

Ans:

The two wave speeds can be written as

$$\upsilon_1 = \sqrt{T_1/\mu} \text{ and } \upsilon_2 = \sqrt{T_2/\mu}$$

Since μ is constant, $\mu = \dfrac{T_2}{\upsilon_2^2} = \dfrac{T_1}{\upsilon_1^2}$, and

$$T_2 = \left(\frac{\upsilon_2}{\upsilon_1}\right)^2 T_1 = \left(\frac{30.0\,\mathrm{m/s}}{20.0\,\mathrm{m/s}}\right)^2 (6.00\,\mathrm{N}) = \boxed{13.5\,\mathrm{N}}$$