St．ID： $\qquad$ ，

Name：
Note：You can use pencil or any pen in answering the problems．Dictionary，calculators and mathematics tables are allowed．Please hand in both solution and this problem sheet．

## ABSOLUTELY NO CHEATING！

## Problems（total 4 problems，120\％）

1．A uniform beam of length 7.60 m and weight $4.50 \times 10^{2} \mathrm{~N}$ is carried by two workers，Sam and Joe，as shown in Figure P12．11．Determine the force that each person exerts on the beam． （30\％）
Ans：
Since the beam is in equilibrium，we choose the center as our pivot point and require that

$$
\Sigma \tau_{\text {center }}=-F_{\mathrm{Sam}}(2.80 \mathrm{~m})+F_{\mathrm{Joe}}(1.80 \mathrm{~m})=0
$$

Or

$$
F_{\mathrm{Joe}}=1.56 F_{\mathrm{Sam}}
$$

Also，


Figure P12．11

$$
\begin{equation*}
\Sigma F_{y}=0 \Rightarrow F_{\mathrm{Sam}}+F_{\mathrm{Joe}}=450 \mathrm{~N} \tag{2}
\end{equation*}
$$

Substitute equation［1］into［2］to get the following

$$
F_{\mathrm{Sam}}+1.56 F_{\mathrm{Sam}}=450 \mathrm{~N} \text { or } F_{\mathrm{Sam}}=\frac{450 \mathrm{~N}}{2.56}=176 \mathrm{~N}
$$

Then，equation［1］yields $F_{\text {Joe }}=1.56(176 \mathrm{~N})=274 \mathrm{~N}$
Sam exerts an upward force of 176 N ．
Joe exerts an upward force of 274 N ．

2．The Explorer VIII satellite，placed into orbit November 3，1960，to investigate the ionosphere， had the following orbit parameters：perigee， 459 km ；apogee， 2289 km （both distances above the Earth＇s surface）；period， 112.7 min ．Find the ratio $v_{p} / v_{a}$ of the speed at perigee to that at apogee．（30\％）
Ans：
By conservation of angular momentum for the satellite，$r_{p} v_{p}=r_{a} v_{a}$ ，or

$$
\frac{v_{p}}{v_{a}}=\frac{r_{a}}{r_{p}}=\frac{2289 \mathrm{~km}+6.37 \times 10^{3} \mathrm{~km}}{459 \mathrm{~km}+6.37 \times 10^{3} \mathrm{~km}}=\frac{8659 \mathrm{~km}}{6829 \mathrm{~km}}=1.27
$$

We do not need to know the period．
3. The spring of the pressure gauge shown in Figure P14.7 has a force constant of $1250 \mathrm{~N} / \mathrm{m}$, and the piston has a diameter of 1.20 cm . As the gauge is lowered into water in a lake, what change in depth causes the piston to move in by 0.750 cm ? (30\%)
Ans:
Assuming the spring obeys Hooke's law, the increase in force on the piston required to compress the spring an additional amount $\Delta x$ is

$$
\Delta F=F-F_{0}=\left(P-P_{0}\right) A=k(\Delta x)
$$

The gauge pressure at depth $h$ beneath the surface of a fluid is

$$
P-P_{0}=\rho g h
$$



Figure P14.7
so we have

$$
\rho g h A=k(\Delta x)
$$

or the required depth is

$$
h=k(\Delta x) \rho g \mathrm{~A}
$$

If $k=1250 \mathrm{~N} / \mathrm{m}, A=\pi d^{2} / 4, d=1.20 \times 10^{-2} \mathrm{~m}$, and the fluid is water $\left(\rho=1.00 \times 10^{3}\right.$ $\mathrm{kg} / \mathrm{m}^{3}$ ), the depth required to compress the spring an additional $\Delta x=0.750 \times 10^{-2} \mathrm{~m}$ is $h=8.46 \mathrm{~m}$
$h=8.46 \mathrm{~m}$
4. A block of unknown mass is attached to a spring with a spring constant of $6.50 \mathrm{~N} / \mathrm{m}$ and undergoes simple harmonic motion with an amplitude of 10.0 cm . When the block is halfway between its equilibrium position and the end point, its speed is measured to be $30.0 \mathrm{~cm} / \mathrm{s}$. Calculate (a) the mass of the block, (10\%) (b) the period of the motion, (10\%) and (c) the maximum acceleration of the block. ( $10 \%$ )
Ans:
(a) Energy is conserved for the block-spring system between the maximum-displacement and the half-maximum points:

$$
\begin{aligned}
& (K+U)_{i}=(K+U)_{f} \\
& 0+\frac{1}{2} k A^{2}=\frac{1}{2} m v^{2}+\frac{1}{2} k x^{2} \\
& \frac{1}{2}(6.50 \mathrm{~N} / \mathrm{m})(0.100 \mathrm{~m})^{2}=\frac{1}{2} m(0.300 \mathrm{~m} / \mathrm{s})^{2} \\
& \\
& +\frac{1}{2}(6.50 \mathrm{~N} / \mathrm{m})\left(5.00 \times 10^{-2} \mathrm{~m}\right)^{2} \\
& 3.25 \times 10^{-2} \mathrm{~J}=\frac{1}{2} m(0.300 \mathrm{~m} / \mathrm{s})^{2}+8.12 \times 10^{-3} \mathrm{~J} \\
& \text { giving } m=\frac{2\left(2.44 \times 10^{-2} \mathrm{~J}\right)}{9.0 \times 10^{-2} \mathrm{~m}^{2} / \mathrm{s}^{2}}=0.542 \mathrm{~kg}
\end{aligned}
$$

(b)

$$
\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{6.50 \mathrm{~N} / \mathrm{m}}{0.542 \mathrm{~kg}}}=3.46 \mathrm{rad} / \mathrm{s}
$$

Then, $\quad T=\frac{2 \pi}{\omega}=\frac{2 \pi \mathrm{rad}}{3.46 \mathrm{rad} / \mathrm{s}}=1.81 \mathrm{~s}$
(c) $a_{\max }=A \omega^{2}=(0.100 \mathrm{~m})(3.46 \mathrm{rad} / \mathrm{s})^{2}=1.20 \mathrm{~m} / \mathrm{s}^{2}$

