Chapter 11

1. The displacement vectors 42.0 cm at 15.0° and 23.0 cm at 65.0° both start from the origin and form two sides of a parallelogram. Both angles are measured counterclockwise from the x axis. (a) Find the area of the parallelogram. (b) Find the length of its longer diagonal.

   Ans:

   (a)
   $$\text{area} = \left|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right| = AB\sin\theta = (42.0\text{ cm})(23.0\text{ cm})\sin(65.0° - 15.0°)$$
   $$= \boxed{740\text{ cm}^2}$$

   (b) The longer diagonal is equal to the sum of the two vectors.
   $$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \left[(42.0\text{ cm})\cos 15.0° + (23.0\text{ cm})\cos 65.0°\right]\hat{\mathbf{i}}$$
   $$+\left[(42.0\text{ cm})\right]\sin 15.0° + (23.0\text{ cm})\sin 65.0°]\hat{\mathbf{j}}$$
   $$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (50.3\text{ cm})\hat{\mathbf{i}} + (31.7\text{ cm})\hat{\mathbf{j}}$$
   $$\text{length} = \left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \sqrt{(50.3\text{ cm})^2 + (31.7\text{ cm})^2} = \boxed{59.5\text{ cm}}$$

2. A uniform solid disk of mass m = 3.00 kg and radius r = 0.200 m rotates about a fixed axis perpendicular to its face with angular frequency 6.00 rad/s. Calculate the magnitude of the angular momentum of the disk when the axis of rotation (a) passes through its center of mass and (b) passes through a point midway between the center and the rim.

   Ans:

   (a) For an axis of rotation passing through the center of mass, the magnitude of the angular momentum is given by
   $$L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}(3.00\text{ kg})(0.200\text{ m})^2(6.00\text{ rad/s})$$
   $$= \boxed{0.360\text{ kg}\cdot\text{m}^2/\text{s}}$$

   (b) For a point midway between the center and the rim, we use the parallel-axis theorem to find the moment of inertia about this point. Then,
   $$L = I\omega = \left[\frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2\right]\omega$$
   $$= \frac{3}{4}(3.00\text{ kg})(0.200\text{ m})^2(6.00\text{ rad/s}) = \boxed{0.540\,\text{kg}\cdot\text{m}^2/\text{s}}$$

3. A light, rigid rod of length $\ell = 1.00$ m joins two particles, with masses $m_1 = 4.00$ kg and $m_2 = 3.00$ kg, at its ends. The combination rotates in the xy plane about a pivot through the center of the rod (Fig. P11.11). Determine the angular momentum of the system about the origin when the speed of each particle is 5.00 m/s.

Ans:

Taking the geometric center of the compound object to be the pivot, the angular speed and the moment of inertia are

$$\omega = v/r = (5.00\text{ m/s})/0.500\text{ m} = 10.0\text{ rad/s}$$

and

$$I = \Sigma mr^2 = (4.00\text{ kg})(0.500\text{ m})^2 + (3.00\text{ kg})(0.500\text{ m})^2 = 1.75\text{ kg}\cdot\text{m}^2$$

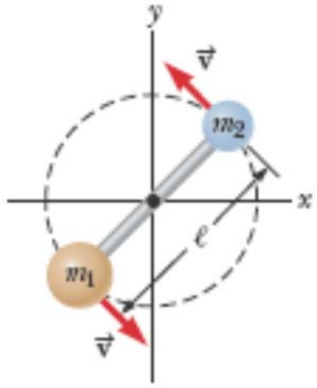

Figure P11.11

By the right-hand rule, we find that the angular velocity is directed out of the plane. So the object's angular momentum, with magnitude

$L = I\omega = (1.75\text{ kg m}^2)(10.0\text{ rad/s})$ is the vector

$$\boxed{\vec{\mathbf{L}} = \left(17.5\text{ kg m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

4. A disk with moment of inertia I_1 rotates about a frictionless, vertical axle with angular speed ω_i. A second disk, this one having moment of inertia I_2 and initially not rotating, drops onto the first disk (Fig. P11.30). Because of friction between the surfaces, the two eventually reach the same angular speed ω_f. (a) Calculate ω_f. (b) Calculate the ratio of the final to the initial rotational energy.

Ans:

(a) From conservation of angular momentum for the isolated system of two disks:

$$(I_1 + I_2)\omega_f = I_1\omega_i \quad \text{or} \quad \omega_f = \boxed{\frac{I_1}{I_1 + I_2}\omega_i}$$

This is an example of a totally inelastic collision.

(b) $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$ and $K_i = \frac{1}{2}I_1\omega_i^2$

so $$\frac{K_f}{K_i} = \frac{\frac{1}{2}(I_1 + I_2)}{\frac{1}{2}I_1\omega_i^2}\left(\frac{I_1}{I_1 + I_2}\omega_i\right)^2 = \boxed{\frac{I_1}{I_1 + I_2}}$$

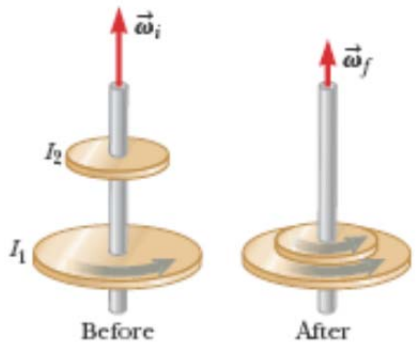

Figure P11.30

5. A spacecraft is in empty space. It carries on board a gyroscope with a moment of inertia of $I_g = 20.0\ \text{kg} \cdot \text{m}^2$ about the axis of the gyroscope. The moment of inertia of the spacecraft around the same axis is $I_s = 5.00 \times 10^5\ \text{kg} \cdot \text{m}^2$. Neither the spacecraft nor the gyroscope is originally rotating. The gyroscope can be powered up in a negligible period of time to an angular speed of 100 rad/s. If the orientation of the spacecraft is to be changed by 30.0°, for what time interval should the gyroscope be operated?

Ans:

Angular momentum of the system of the spacecraft and the gyroscope is conserved. The gyroscope and spacecraft turn in opposite directions.

$$0 = I_1\omega_1 + I_2\omega_2 : \qquad -I_1\omega_1 = I_2\frac{\theta}{t}$$

$$\left(-20\ \text{kg}\cdot\text{m}^2\right)\left(-100\ \text{rad/s}\right) = (5\times10^5\,\text{kg}\cdot\text{m}^2)\left(\frac{30°}{t}\right)\left(\frac{\pi\ \text{rad}}{180°}\right)$$

$$t = \frac{2.62\ \times\ 10^5\text{s}}{2000} = \boxed{131\text{s}}$$