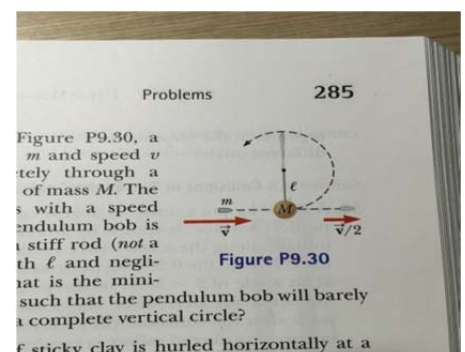
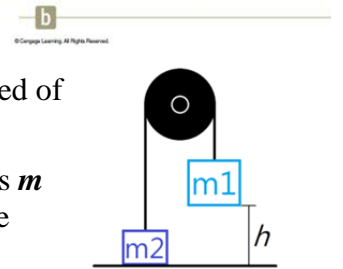
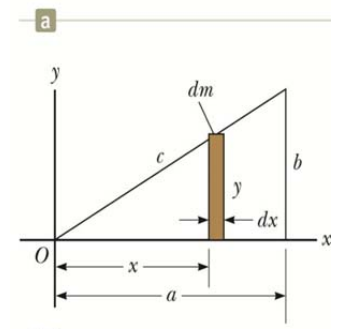
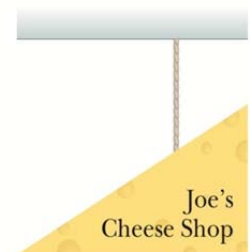
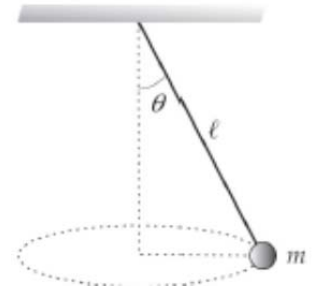


SN: _____, Name: _____

*Note: You can use pencil or any pen in answering the problems.
Dictionary, calculators and mathematics tables **are** allowed.
Please hand in both solution and this problem sheet.
ABSOLUTELY NO CHEATING!*

Problems (total 5 problems, 100%)

- Angular momentum-1:** (20%) A conical pendulum consists of a bob of mass m in motion in a circular path in a horizontal plane as shown in the figure to the right. During the motion, the supporting wire of length l maintains a constant angle θ with the vertical. Show that the magnitude of the angular momentum of the bob about the vertical dashed line is $L = \left(\frac{m^2 g l^3 \sin^4 \theta}{\cos \theta}\right)^{1/2}$.
- Center of Mass:** (20%) If you were to hang a triangular metal sign from a vertical string as shown in the figure to the right. The bottom of the sign is to be parallel to the ground. At what distance from the left end of the sign should you attach the support string?
- Newton's law:** (20%) A block of mass m is dropped from the fourth floor of an office building and hits the sidewalk below at speed v . From what floor should the block be dropped to double that impact speed?
- Newton's law:** (20%) Two objects are connected by a light string passing over a light, frictionless pulley as shown in figure above. The object of mass $m_1 = 5.00$ kg is released from rest at a height $h = 4.00$ m above the table. Using the isolated system model, Determine the speed of the object of mass $m_2 = 3.00$ kg just as the 5.00-kg object hits the table.
- Energy conservation:** (20%) As shown in Figure 9.30, a bullet of mass m and speed v passes completely through a pendulum bob of mass M . The bullet emerges with a speed of $\frac{v}{2}$. The pendulum bob is suspended by a stiff rod (not a string) of length l and negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle?



Midterm-1 Solution

1

We start with the particle under a net force model in the x and y directions:

$$\sum F_x = ma_x: \quad T \sin \theta = \frac{mv^2}{r}$$

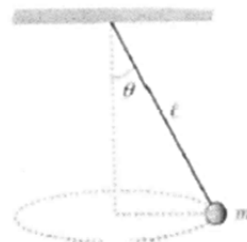
$$\sum F_y = ma_y: \quad T \cos \theta = mg$$

So $\frac{\sin \theta}{\cos \theta} = \frac{v^2}{rg}$ and $v = \sqrt{rg \frac{\sin \theta}{\cos \theta}}$

then $L = rmv \sin 90.0^\circ = rm \sqrt{rg \frac{\sin \theta}{\cos \theta}} = \sqrt{m^2 g r^3 \frac{\sin \theta}{\cos \theta}}$

and since $r = \ell \sin \theta$,

$$L = \sqrt{m^2 g \ell^3 \frac{\sin^4 \theta}{\cos \theta}}$$



2. (a) A triangular sign to be hung from a single string. (b) Geometric construction for locating the center of mass.

Evaluate dm :

$$dm = \rho y t dx = \left(\frac{M}{\frac{1}{2}abt} \right) y t dx = \frac{2My}{ab} dx$$

Use Equation 9.32 to find

the x coordinate of the

center of mass:

$$(1) \quad x_{\text{CM}} = \frac{1}{M} \int x dm = \frac{1}{M} \int_0^a x \frac{2My}{ab} dx = \frac{2}{ab} \int_0^a xy dx$$

To proceed further and evaluate the integral, we must express y in terms of x . The line representing the hypotenuse of the triangle in Figure 9.20b has a slope of b/a and passes

through the origin, so the equation of this line is $y = (b/a)x$.

Substitute for y in Equation (1):

$$x_{\text{CM}} = \frac{2}{ab} \int_0^a x \left(\frac{b}{a} x \right) dx = \frac{2}{a^2} \int_0^a x^2 dx = \frac{2}{a^2} \left[\frac{x^3}{3} \right]_0^a$$

$$= \frac{2}{3} a$$

3.

$$\therefore V^2 = V_0^2 + 2aS \quad \therefore V^2 = 0 + 2 \cdot g \cdot 4 \text{ (floor)}, (2V)^2 = 0 + 2 \cdot g \cdot K$$

Thus the algebra K is for 16

4. Assume $g = 10 \text{ m/s}^2$,

$$- (5 + 3) \times a = (5 - 3) \times 10, a = 2.5 \text{ m/s}^2$$

$$V^2 = V_0^2 + 2as$$

$$= 0 + 2 \times 2.5 \times 4$$

$$= 20$$

$$V = 2\sqrt{5} \text{ m/s}$$

$$5. \frac{1}{2} Mv'^2 = Mg \cdot 2l, v' = \sqrt{4gl}$$

$$mv = M \cdot \sqrt{4gl} + m \cdot \frac{1}{2} v$$

$$v = \frac{4M}{m} \sqrt{gl}$$

SN: _____, Name: _____

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are allowed.

The followings are some useful mathematics you may use without proof in answering your problems.

$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots$. Time average $\overline{x(t)^n} = \langle x(t)^n \rangle = \frac{1}{T} \int_0^T x(t)^n dt$

For a second order differential equation, $\frac{d^2x}{dt^2} + ax = 0$, the general solution of this equation is

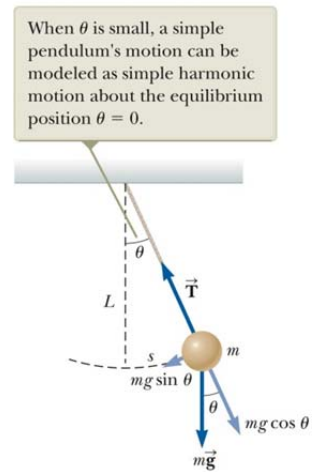
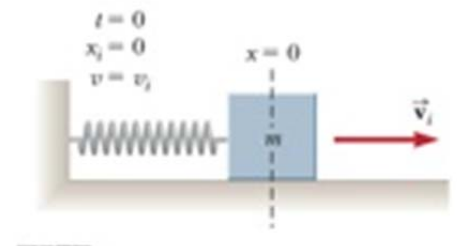
$x(t) = x_0 \cos(at + \phi)$, where x_0 is the maximum, and ϕ is the phase angle.

$N_A = 6 \times 10^{23}$, $R = \text{Gas constant} = 8.31 \text{ J/mole K}$, room temperature = 300K, 1 atm = $1.01 \times 10^5 \text{ Pa}$.

$\overline{v_x} = \frac{v_{rms}}{\sqrt{3}}$ for ideal gas.

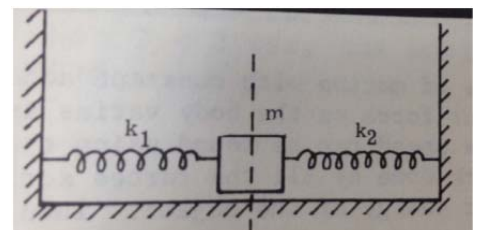
Problems (7 Problems, total 130%)

- 1. Harmonic Oscillation (20%):** (a, b) Refer to the figure to the right, write down differential equations that can describe the motion for a mass m attached to a spring of force constant k and the same mass attached to a pendulum of length L . In both cases, use the given parameters. In the upper case, describe the motion in terms of its displacement x ; while in the lower case, describe its motion in terms of the angle θ . (c) If we treat the pendulum as a simple harmonic oscillator and look at only the horizontal displacement s at small angle, what will the equation look like?



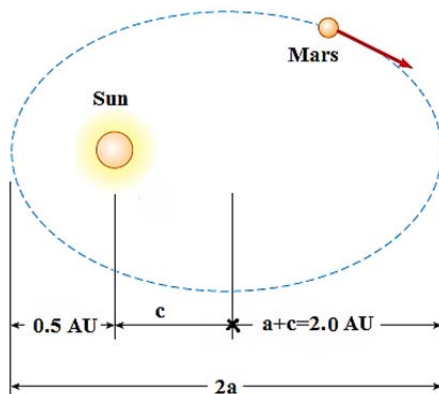
- 2. Entropy and thermodynamics (20%):** In an air conditioning process a room is kept at 290°K while the temperature outside is 305°K. The refrigerating machine has compression cylinders operating at 320°K (located outside) and expansion coil inside the house operating at 280°K. If the machine operates reversibly, how much work must be done for each transfer of 5000 joules of heat from the house? What is the entropy changes occur outside the house for this amount of refrigeration?

- 3. Spring system (20%):** For a spring system show to the right, calculating the frequency of oscillation of the configuration shown in the figure. All surfaces are



frictionless.

4. **Adiabatic Process (15%):** Prove that for an adiabatic expansion of an ideal gas, $PV^\gamma = \text{constant}$, where $\gamma = C_p/C_v$.
5. **Standing wave (20%):** In an area confined by two walls, similar to the block in the last problem confined between two walls. If you send a right traveling wave from the left wall and a left traveling wave from the right wall, both waves are identical except the traveling direction. You could generate a standing wave. (a) What is the possible wave function of this standing wave? (b) Why it is a standing wave?
6. **Pressure (15%):** (a) Suppose you are driving a submarine in Pacific Ocean, calculate the absolute pressure at an ocean depth of 2.0 km. Let the density of seawater is 1000kg/m^3 and the air above exerts a pressure of 100.0 kPa. (b) In this depth, what pressure/ force will be exerted by the water on a circular window of radius 2.5 m of the submarine? [20+10 = 30]
7. **Kepler's law (20%):** (a) Write down the Kepler's third law of planetary motion. (b) Let the Mars is moving in an elliptical orbit around the Sun. Its distance from the Sun ranges between 0.5 AU to 2.5 AU. Calculate the eccentricity of Mars orbit (c) Find out the period of it around the sun. (Here, 1 AU= one astronomical unit, the average distance from Sun to Earth = 1.496×10^{11} m and $K_s = 2.97 \times 10^{-19} \text{ s}^2/\text{m}^3$).



Quiz-Final-1

1. Solution:

$$(a) F = -kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

$$a_x = \frac{dV_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\therefore \frac{d^2x(t)}{dt^2} = -\frac{k}{m}x$$

$$\therefore \frac{d^2x(t)}{dt^2} + \frac{k}{m}x = 0$$

$$(b) F_T = -mg \sin \theta = mas = m \frac{d^2S}{dt^2}$$

$$\text{Where, } S = L\theta \quad \therefore \frac{d^2S}{dt^2} = L \frac{d^2\theta}{dt^2}$$

$$\Rightarrow -mg \sin \theta = mL \frac{d^2\theta}{dt^2}$$

$$\therefore \frac{d^2\theta}{dt^2} = -\left(\frac{g}{L}\right) \sin \theta$$

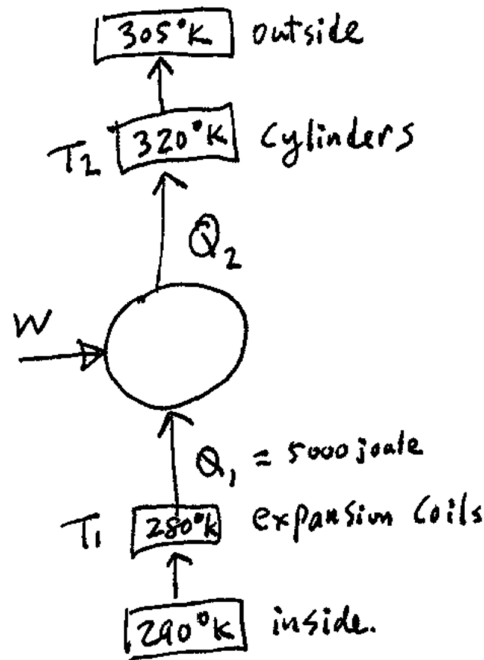
$$(c) \text{ In this figure, } S=R\theta, \therefore \theta = \frac{S}{R} \Rightarrow \ddot{\theta} = \frac{\ddot{S}}{R}$$

$$\ddot{\theta} + \left(\frac{g}{L}\right)\theta = 0$$

$$\frac{\ddot{S}}{R} + \left(\frac{g}{L}\right)\frac{S}{R} = 0$$

$\therefore \ddot{S} + \left(\frac{g}{L}\right)S = 0$, this is the same as in (a). So they both are Simple Harmonic Oscillators

2. Solution:



(a) Schemically, all components are showing at T_4 above.
 The efficiency η of a reversible heat engine

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1} \quad \text{--- heating}$$

$$\eta = \frac{W}{Q_2} = \frac{Q_2 - Q_1}{Q_2} = \frac{T_2 - T_1}{T_2} \quad \text{--- cooling}$$

$$\therefore \text{for cooling } \eta = 1 - \frac{280}{320} = 0.125 = \frac{Q_2 - Q_1}{Q_2} = \frac{Q_2 - 5000}{Q_2}$$

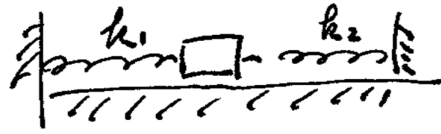
$$\therefore Q_2 = 5715 \text{ J} \Rightarrow W = \eta \cdot Q_2 = 0.125 \times 5715 \approx 715 \text{ J}$$

(b)

$$\Delta S = \frac{Q_2}{T_{\text{out}}} - \frac{Q_2}{T_{\text{cylinder}}} = Q_2 \left[\frac{1}{305} - \frac{1}{320} \right]$$

$$= 0.88 \text{ J/K}$$

3. Solution:



$$\Delta x_1 = -\Delta x_2.$$

If we generate a displacement Δx , then

$$\Delta x_1 = \Delta x$$

$$\Delta x_2 = -\Delta x$$

$$\begin{aligned} F &= -k_1 \Delta x_1 - (-k_2 \Delta x_2) = -k_1 \Delta x + k_2 (-\Delta x) \\ &= -(k_1 + k_2) \Delta x \\ &= -K' \Delta x, \quad K' = k_1 + k_2. \end{aligned}$$

$$V = \frac{1}{2\pi} \sqrt{\frac{K'}{m}} = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}}$$

4. Solution:

Adiabatic process for an ideal gas

$$Q = 0.$$

$$\Delta E_{\text{int}} = Q + W = \overset{W}{Q} = n C_v dT = -P dV$$

$$\text{But } PV = nRT$$

$$\begin{aligned} P dV + V dP &= nR dT \\ &= -\frac{R}{C_v} P dV \end{aligned}$$

$$\begin{aligned} \frac{dV}{V} + \frac{dP}{P} &= -\left(\frac{C_p - C_v}{C_v}\right) \frac{dV}{V} \\ &= (1 - \gamma) \frac{dV}{V} \end{aligned}$$

$$\frac{dP}{P} + \gamma \frac{dV}{V} = 0$$

$$\ln P + \gamma \ln V = \text{Constant}$$

$$PV^\gamma = \text{Constant}$$

5. Solution:



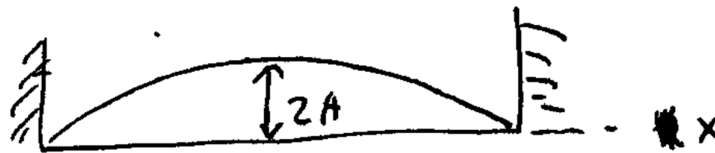
Let $y_1 = A \sin(kx - \omega t)$ traveling to the right
 $y_2 = A \sin(kx + \omega t)$ traveling to the left

The resulting wave is

$$(a) \quad y = y_1 + y_2 = A \sin(kx - \omega t) + A \sin(kx + \omega t) \\ = \underline{2A \sin(kx) \cos(\omega t)}$$

(b) You have a new wave

$$y = \underbrace{2A \sin(kx)}_{\text{Amplitude}} \underbrace{\cos(\omega t)}_{\text{Oscillation}}$$



The wave form oscillates as a function of $\cos(\omega t)$. It is not traveling anymore it stands there, so it is called Standing Wave.

6. Solution :

(a) The absolute pressure is

$$P = P_0 + h\rho g, \text{ Where } P_0 \text{ is the air pressure and } h = 2.0\text{km}, \rho = 1000\text{kg} / \text{m}^3, g = 9.8\text{m} / \text{s}^2$$

$$\therefore P = 100000\text{Pa} + (2000\text{m} \times 1000\text{kg} / \text{m}^3 \times 9.8\text{m} / \text{s}^2) = 1.97 \times 10^7 \text{Pa}$$

(b) The gauge pressure is the difference in pressure between the water outside and the air inside the submarine, which we suppose is at 1.00 atmosphere.

$$\therefore P_{\text{gauge}} = P - P_0 = h\rho g = 1.96 \times 10^7 \text{Pa}$$

The resultant inward force on the window is then

$$F = P_{\text{gauge}} A = 1.96 \times 10^7 \text{Pa} \times \pi r^2 = 1.96 \times 10^7 \text{Pa} \times 3.14 \times (2.0)^2 = 2.5 \times 10^8 \text{N}$$

7. Solution:

(a) Kepler's third law of planetary motion is

$$T^2 = K_s a^3, \text{ where } T \text{ is period of planetary revolution, } a = \text{The semimajor axis length of orbit} \\ \text{and } K_s = \text{constant of proportionality (with respect to Sun)}$$

(b) From figure we can find the major axis length of the orbit is

$$2a = 2.5 \therefore a = 1.25 \text{ AU}$$

and $a+c=2.0 \therefore c=0.75 \text{ AU}$, where c is the distance between focus and the center of orbital axis

$$\therefore \text{eccentricity } e = \frac{c}{a} = \frac{0.75}{1.25} = 0.66$$

(c) We know that $T^2 = K_s a^3$

$$\text{So } T = \sqrt{K_s a^3} = a \sqrt{K_s a} = 1.25 \text{ AU} \times \sqrt{2.97 \times 10^{-19} \times 1.25 \text{ AU}} \\ = 1.25 \times 1.496 \times 10^{11} \times \sqrt{2.97 \times 10^{-19} \times 1.25 \times 1.496 \times 10^{11}} \\ = 55165000 \text{ seconds} = \frac{55165000 \text{ days}}{3600 \times 24} = 638.5 \text{ days} = 1.78 \text{ years}$$