1.

(a)

$$e \equiv \frac{W_{Carnot}}{|Q_h|} = \frac{|Q_h|-|Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$$

In a Carnot cycle, $\Delta E_{in} = 0$ (path $A \to B$)

$\therefore |Q_h| = |-W_{AB}|$ $\quad W_{AB} \equiv$ work done between $A \to B$

$A \to B$, $|Q_h| = |-W_{AB}| = nRT_h \ln\left(\frac{V_B}{V_A}\right)$ absorb energy

$C \to D$, $|Q_c| = |-W_{CD}| = nRT_c \ln\left(\frac{V_C}{V_D}\right)$

$$\therefore \frac{|Q_c|}{|Q_h|} = \frac{T_c}{T_h} \frac{\ln\left(\frac{V_C}{V_D}\right)}{\ln\left(\frac{V_B}{V_A}\right)} \quad — ①$$

But $P_i V_i^{\gamma} = P_f V_f^{\gamma} \rightarrow T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$

$$\left.\begin{array}{ll} B \to C & T_h V_B^{\gamma-1} = T_c V_C^{\gamma-1} \\ D \to A & T_h V_A^{\gamma-1} = T_c V_D^{\gamma-1} \end{array}\right\} \Rightarrow \left(\frac{V_B}{V_A}\right)^{\gamma-1} = \left(\frac{V_C}{V_D}\right)^{\gamma-1} \quad \therefore \frac{V_B}{V_A} = \frac{V_C}{V_D} \quad — ②$$

From ① and ② $\quad \frac{|Q_c|}{|Q_h|} = \frac{T_c}{T_h}$

Therefore $\quad e_{Carnot} = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{T_c}{T_h}$ #

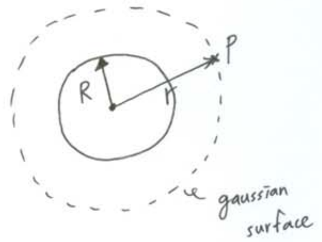
(b) For an engine to have perfect performance $e = 1$

$\Rightarrow |Q_c| = 0$ or $T_c \to 0$ #

(c) The perfect engine requires $T_c = 0$ not possible or $Q_c = 0$, that means it will not give up or waste any energy. This is not possible in reality.

2.

(1) A point outside the sphere : Pick a gaussian surface bigger than the surface of the sphere



$$\boxed{\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}}$$

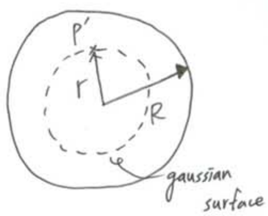
$$\Rightarrow E\oint dA = E\cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{r^2}$$

$$= k_e\frac{Q}{r^2} \quad (r>R) \qquad \propto \frac{1}{r^2}$$

#

(2) A point inside the sphere : Pick a gaussian surface as shown



$$\boxed{\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}}$$

$$\Rightarrow E\cdot 4\pi r^2 = \frac{q_{in}}{\epsilon_0}, \quad q_{in} = \rho\cdot V$$

$$= \frac{Q}{\frac{4}{3}\pi R^3}\cdot\frac{4}{3}\pi r^3$$

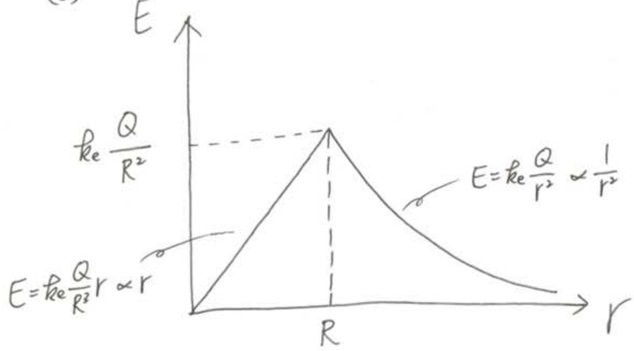
$$= Q\,\frac{r^3}{R^3}$$

$$\therefore E = \frac{1}{4\pi\epsilon_0}\,\frac{Q}{R^3}\,r$$

$$= k_e\frac{Q}{R^3}\,r \quad (r<R) \qquad \propto r$$

#

(3)



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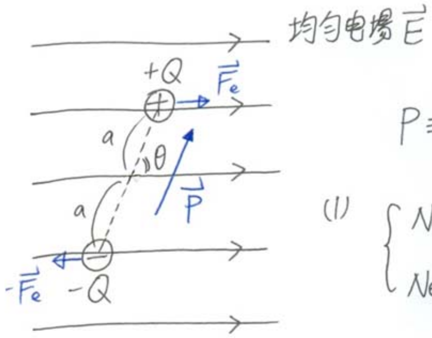
3.



$$P \equiv (2a)\,Q$$

(1) Net force is zero.

Net torque is not zero. (The two forces produce a net torque on this dipole.)

$$\Rightarrow \boxed{\vec{\tau} \equiv \vec{r} \times \vec{F}}$$

$$\Rightarrow \tau = 2Fa\sin\theta$$

$$\underset{F=QE}{=} 2QEa\sin\theta \underset{P=2aQ}{=} PE\sin\theta$$

$$\Rightarrow \vec{\tau} = \vec{P} \times \vec{E} \quad \#$$

(2) The work done to rotate the dipole is stored as potential energy in the system.

$$dW = \tau\, d\theta$$

$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau\, d\theta = \int_{\theta_i}^{\theta_f} PE\sin\theta\, d\theta = PE \underbrace{\int_{\theta_i}^{\theta_f} \sin\theta\, d\theta}_{-\cos\theta \big|_{\theta_i}^{\theta_f}} = PE(\cos\theta_i - \cos\theta_f) \quad \#$$

(若令 $U_i = 0$ at $\theta_i = 90^\circ$，則 $\cos\theta_i = 0$ & $U = U_f = -\vec{P}\cdot\vec{E}$)

4.|

4. The isolated system always tend toward disorder.

① Entropy is a measurent of the degree of the disorder of a system.

It is defined by

$$\Delta S = \int \frac{dQ}{T}$$

② For a adiabatic free expansion of a gas, if gas initial volume V_i expands to V_f. Then the changes in entropy can be defined as

$$\Delta S = \int_i^f \frac{dQ_r}{T} = \frac{1}{T}\int_i^f dQ_r$$

Since T is constant for this process, so we can't take $Q=0$ (even for adiabatic case $Q=0$), so we must choice an isothermal, reversible expansion here. In isothermal process

$$dQ = -dW = -PdV$$

therfore,

$$\Delta S = -\int_{V_i}^{V_f} \frac{P}{T}\, dV = \frac{P}{T}\int_{V_i}^{V_f} dV$$

Let V_f is lage than V_i

So $\Delta S =$ positive

$$= \frac{nR}{V}\int_{V_i}^{V_f} dV = nR\ln\left(\frac{V_f}{V_i}\right)$$

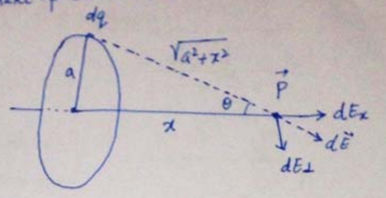
For an ideal gas

$PV = nRT$

$$\boxed{\Delta S = nR\ln\left(V_f/V_i\right).}$$

5.

Q.5 (1) Let the direction of electric field is in x- direction.
For the symmetry the perpendicular component of electric field will be canceled each other.



The parallel component of an electric field contribution from a segment of charge 'dq' on the ring

$$dE_x = k_e \frac{dq}{r^2} \cos\theta \quad \ldots \text{(i)}$$

where $k_e = \frac{1}{4\pi\epsilon_0}$

$$= \frac{dq}{\sqrt{a^2+x^2}}$$

From the geometry, we can write

$$\cos\theta = \frac{x}{\sqrt{a^2+z^2}} \quad \ldots \text{(ii)}$$

From eq. (i) and (ii) we get

$$dE_x = k_e \frac{x\,dq}{(a^2+x^2)\sqrt{a^2+x^2}} = \frac{k_e x}{(a^2+x^2)^{3/2}} dq$$

So, from all segments of the ring, the electric field at P is

$$E_x = \int \frac{k_e x}{(a^2+x^2)^{3/2}} dq \quad \text{or} \quad \boxed{E = \frac{k_e x}{(a^2+x^2)^{3/2}} Q}$$

(2) If $x \gg a$, then the ring will act as a point charge
so we can write $a = 0$,

$$E = \frac{K_e x\, Q}{x^3} = K_e \frac{Q}{x^2}$$

$$\boxed{E = K_e \frac{Q}{x^2}}$$