General Physics I, Midterm Exam 1 solution

1. This problem is from Page 201 (Example 7.9) of text book.
(a) The separation of two atoms is where the potential is in its minimum. To find the minimum, we set $\frac{dU(x)}{dx} = 4\varepsilon \frac{d}{dx}\left[(\frac{\sigma}{x})^{12} - (\frac{\sigma}{x})^{6} \right] = 4\varepsilon \left[\frac{-12\sigma^{12}}{x^{13}} + \frac{6\sigma^{6}}{x^{7}} \right] = 0$

$$\frac{dU(x)}{dx} = 4\varepsilon \frac{d}{dx}\left[(\frac{\sigma}{x})^{12} - (\frac{\sigma}{x})^{6} \right] = 4\varepsilon \left[\frac{-12\sigma^{12}}{x^{13}} + \frac{6\sigma^{6}}{x^{7}} \right] = 0, \quad x = (2)^{\frac{1}{6}}\sigma$$

(b) Plug in numbers given, $x=2.95\times10^{-10}$ m
(c) The potential energy curve is shown in page 189 of the text book.
(d) When $x= 4.5\times10^{-10}$ m, the two atoms are subject to a restoration form to bring them together to the equilibrium point ($x=2.95\times10^{-10}$ m)
(e) This can be proved by taking the first derivative of the potential, $\frac{dU}{dx}>0$, this is the force of the two atoms at that point, so it is a restoration force to bring them together.

2.

(a) $v = v_0 + at$. $v_0 = 0$

$v = at = [\frac{F}{m}]t = [\frac{100\ \text{dyne}}{20g}] \times 10\text{sec} = 50\ \text{cm/sec}$

$K_E = \frac{1}{2}mv^2 = \frac{1}{2}(20g) \times 50\ \text{cm/sec} = 25000\ \text{ergs}$

(b) To calculate the work. we need to know the distance S travelled

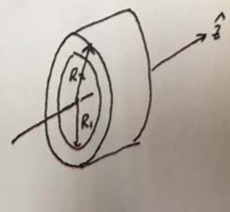
$S = \frac{1}{2}at^2 = \frac{1}{2}(\frac{F}{m})t^2$

$= \frac{1}{2}(\frac{100\ \text{dyne}}{20g}) \times (10\ \text{sec})^2 = 250\ \text{cm}$

$W = (100\ \text{dyne}) \times 250\ \text{cm} = 25000\ \text{ergs}$

3.

(b) Annular cylinder



$I = \int dm\, r^2$

$dm = 2\pi r H \rho\, dr$

$= \int 2\pi r H \rho\, dr\, r^2$

$= 2\pi H \rho \int_{R_1}^{R_2} r^3 dr = 2\pi H \rho \frac{1}{4}(R_2^4 - R_1^4)$

But $M = \pi R_2^2 H \rho - \pi R_1^2 H \rho$

$= \pi H \rho (R_2^2 - R_1^2)$

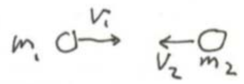
$\therefore I = \frac{\pi}{2} H \rho (R_2^4 - R_1^4) = \frac{\pi}{2} H \rho (R_2^2 + R_1^2)(R_2^2 - R_1^2)$

$= \frac{1}{2} M (R_1^2 + R_2^2)$

4.

Using Newton's 3rd law

$F_{12} = -F_{21}$



$F_{12} + F_{21} = 0$

$m_2 a_2 + m_1 a_1 = 0$

$m_1 \frac{dv_1}{dt} + m_2 \frac{dv_2}{dt} = 0$

$\frac{d(m_1 v_1)}{dt} + \frac{d}{\cancel{dt}}\left(\frac{m_2 v_2}{dt}\right) = 0$

$\Rightarrow \frac{d}{dt}\left(M_1 V_1 + M_2 V_2\right) = 0$

$M_1 V_1 + M_2 V_2 = \text{Constant}$

— Conservation of linear Momentum

5.

Suppose the force is pulling to the $+\hat{x}$ direction, $F_{app} = kx$

$F_{sp}(x) = -kx \Rightarrow$ the force of the spring is in the $-\hat{x}$ direction

the work done by the $\vec{F}$ is

$W_{app} = -W_S = -\int_0^{x_{max}} (-kx)\,dx = \left[\frac{1}{2}kx^2\right]_0^{x_{max}}$

$= \frac{1}{2}kx_{max}^2$

This means this much of energy will be Stored in the spring as elastic potential energy

6.

Solution:

We know that Energy , $E = \text{Power}\times\text{time} = P\times t$

Here given as , P= 400kW=4×10^5 W and t= 10 hrs= 10×3600 s =36000s

so , the total energy will be consumed by the car is

$E = 4\times10^5\times36000 = 1.4\times10^{10}$ J

If we consider the total enery is used to convert kinetic energy

of the car , then we get, $E = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{2E/m} = S/t$, where S= distance, v =velocity

$\Rightarrow S = \sqrt{2E/m}\times t = \sqrt{(2\times1.4\times10^{10}\ \text{J})/250\text{kg}}\times36000\text{s}$

So,distance will be coverd,$S = 3.8\times10^8$ km