

SN: _____, Name: _____

Final 1 Solution

Chapter 12-19, Serway; ABSOLUTELY NO CHEATING!

Please write the answers on the blank space or on the back of this paper to save resources.

1. (a) $F = -kx = ma_x$, $a_x = -\frac{k}{m}x$

$$a_x = \frac{dV_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\therefore \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

(b) $F_T = -mg \sin \theta = ma_s = m \frac{d^2s}{dt^2}$

$$s = L\theta \rightarrow \frac{d^2s}{dt^2} = L \frac{d^2\theta}{dt^2}$$

$$\therefore -mg \sin \theta = mL \frac{d^2\theta}{dt^2}$$

if θ is small, $\sin \theta \approx \theta$

$$\rightarrow -mg\theta = L \frac{d^2\theta}{dt^2}$$

$\therefore \frac{d^2\theta}{dt^2} + \left(\frac{g}{L}\right)\theta = 0$ This is the same as that of the Spring-mass system.

(c) $s = L\theta \therefore \frac{d^2s}{dt^2} = L \frac{d^2\theta}{dt^2}$

The above equation can be rewritten to

$$\frac{1}{L} \frac{d^2s}{dt^2} + \left(\frac{g}{L}\right) \frac{s}{L} = 0 \rightarrow \frac{d^2s}{dt^2} + \left(\frac{g}{L}\right)s = 0$$

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2. (a) Speed of the wave
Use the figure

$$F_r = 2T \sin \theta \approx 2T\theta \text{ if } \theta \text{ is small}$$

$$m = \mu \Delta s = \mu R 2\theta = 2\mu R\theta$$

$$\text{But } F_r = ma = \frac{mv^2}{R} = 2T\theta = \frac{2\mu R\theta v^2}{R}$$

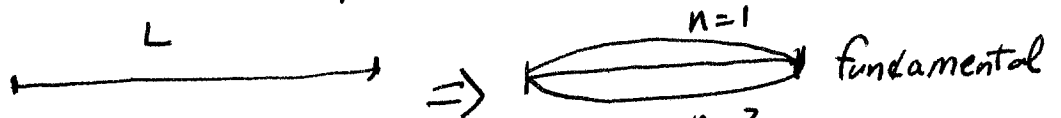
$$\therefore v = \sqrt{\frac{T}{\mu}}$$

(b) $y_1 = A \sin(kx - \omega t)$; $y_2 = A \sin(kx + \omega t)$

$$y_{\text{total}} = y_1 + y_2 = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

$$= \underbrace{2A \sin(kx)}_{\text{New amplitude}} \cos(\omega t) \quad - \textcircled{1}$$

(c)



$$\lambda_n = \frac{2L}{n} \cdot n = 1, 2, 3 \dots$$

$$f_n = \frac{v}{\lambda} = f_n = \frac{v}{\lambda_n} = n \frac{v}{2L}$$

$n = 1, 2, 3$ (or $1, 2, 3$)

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \quad f_1 = \frac{1}{2L} \sqrt{\frac{T}{\mu}}, \quad f_2 = \frac{2}{2L} \sqrt{\frac{T}{\mu}}, \quad f_3 = \frac{3}{2L} \sqrt{\frac{T}{\mu}}$$



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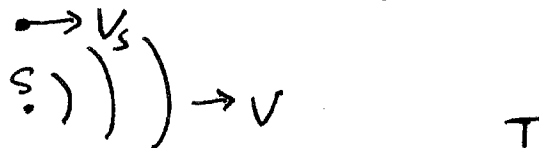
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3. Doppler effect

Sound frequency f , speed v

Now, the detector is stationary, Source moves towards the detector with a speed v_s



During a time period T .

Wave front moves vT . (W_1)

Source moves $v_s T$ (W_2)

In this time period, the two wave fronts (W_1 and W_2)

Separate by a distance $(vT - v_s T)$ and this is

the detected new wave length λ' of the wave due to the moving of the source,

$$\begin{aligned} \therefore f' &= \frac{v}{\lambda'} = \frac{v}{vT - v_s T} \\ &= \frac{v}{\frac{v}{f} - \frac{v_s}{f}} = f \left(\frac{v}{v - v_s} \right) \end{aligned}$$



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4. gravitational force

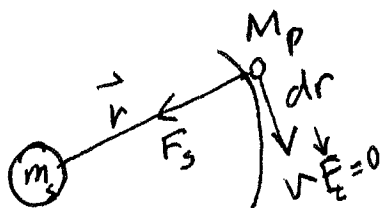
$$F = G \frac{M_1 M_2}{r^2} \quad , \quad G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$\begin{aligned} \rightarrow r &= \sqrt{G \frac{M_1 M_2}{F}} = \sqrt{\frac{6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2} \cdot 2 \times 10^{30} \text{ kg} \times 3 \times 10^{30} \text{ kg}}{6 \times 10^{25} \text{ N}}} \\ &= 2.58 \times 10^8 \text{ m} \end{aligned}$$

5 (a) All planets are in elliptical orbits

2. The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals
3. The square of the orbital period is proportional to the cube of the semi-major axis of the orbit

(b)



$$\vec{\tau} = \vec{r} \times \vec{F} = r \times F(r) \hat{r} = 0 \quad \text{No force in the direction of planet}$$

$$\tau = \frac{dL}{dt} = 0 \rightarrow L = \text{constant}$$

$$L = \vec{r} \times \vec{p} = M_p \vec{r} \times \vec{v} = \text{constant}$$

$$dA = \frac{1}{2} |\vec{r} \times d\vec{r}| = \frac{1}{2} |\vec{r} \times \vec{v} dt| = \frac{L}{2M_p} dt$$

$$\frac{dA}{dt} = \frac{L}{2M_p} = \text{constant}$$

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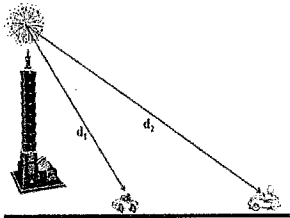
6. The sound intensity at distance d_1 is, suppressing units,

$$I_1 \frac{\Delta P_{\max}^2}{2\rho v} = \frac{(20)^2}{2(1.20)(332)} = 0.5 \text{ W/m}^2$$

If air does not absorb sound energy, the intensity of sound is inversely proportional to the square of the distance from its source. The intensity at distance d_2 is

$$\begin{aligned} I_2 &= \left(\frac{d_1}{d_2}\right)^2 I_1 = \left(\frac{1000\text{m}}{4000\text{m}}\right)^2 I_1 = \frac{1}{16} (0.5 \text{ W/m}^2) \\ &= 3.1 \times 10^{-2} \text{ W/m}^2 \end{aligned}$$

which has an intensity level of



$$\begin{aligned} \beta_2 &= (10 \text{ dB}) \log\left(\frac{I_2}{I_0}\right) = (10 \text{ dB}) \log\left(\frac{3.1 \times 10^{-2} \text{ W/m}^2}{10^{-12} \text{ W/m}^2}\right) \\ &= 100.4 \text{ dB} \end{aligned}$$

Allowing for absorption of the wave over the distance traveled,

$$\beta_2' = \beta_2 - (10 \text{ dB/km})(4000\text{m} - 1000\text{m}) = 74.0 \text{ dB}$$