

SN: _____, Name: _____

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are allowed.

For a second order differential equation, $\frac{d^2x}{dt^2} + ax = 0$, the general solution of this equation is

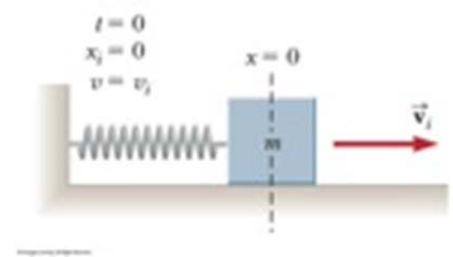
$x(t) = x_0 \cos(at + \phi)$, where x_0 is the maximum, and ϕ is the phase angle.

$N_A = 6 \times 10^{23}$, $R = \text{Gas constant} = 8.31 \text{ J/mole K}$, room temperature = 300K, $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$.

$G = \text{Gravitational constant} = 6.67 \times 10^{-11} \text{ m}^3 \text{ Kg}^{-1} \text{ s}^{-2}$

$\overline{v_x} = \frac{v_{rms}}{\sqrt{3}}$ for ideal gas.

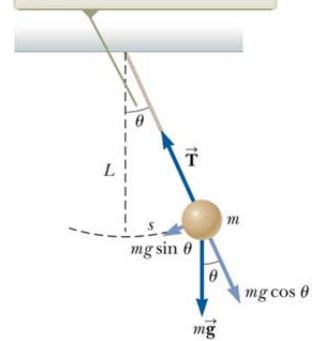
$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$



Problems (6 Problems, total 120%)

- 1. Harmonic Oscillation (20%):** (a, b) Refer to the figure to the right, write down differential equations that can describe the motion for a mass m attached to a spring of force constant k and the same mass attached to a pendulum of length L . In both cases, use the given parameters. In the upper case, describe the motion in terms of its displacement x ; while in the lower case, describe its motion in terms of the angle θ . (c) If we treat the pendulum as a simple harmonic oscillator and look at only the horizontal displacement s at small angle, what will the equation look like?

When θ is small, a simple pendulum's motion can be modeled as simple harmonic motion about the equilibrium position $\theta = 0$.

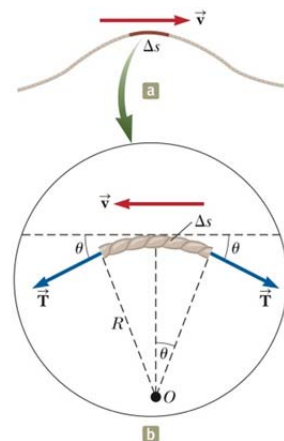


- 2. Speed of waves on String Standing wave (20%):**

(a, 10%) Consider a string as shown in the figure to the right, a string wave is set up in the string and the wave travels to the right (the +x direction). What is the speed of the wave, in terms of the parameters to the right?

(b, 5%) Now if the string were fixed on both ends and we initiate two waves traveling in opposite direction with wave functions can be described as the following:

$$y_1 = A \sin(kx - \omega t)$$



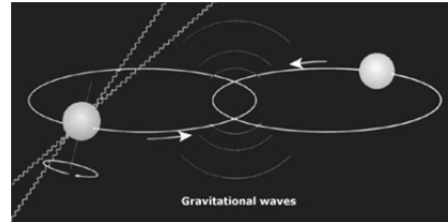
$$y_2 = A \sin(kx + \omega t)$$

What will be the resulting wave, expressed using the given parameters. What is the new amplitude of the combined wave?

(c, 5%) Suppose this string is of length L hanging across the room, what are the frequency (f) of the fundamental and first two harmonics?

3. **Doppler Effect (20%)**: A sound source emits a stable sound of frequency f and sound speed V . A detector is able to detect the sound that can be moving or can be stationary.
 (a) If the detector is stationary, and source is moving toward to the detector with a speed V_s , what will be the detected frequencies?

4. **Gravitational Force (20%)**: The gravitational wave is produced when two star systems merge/collide each other. If the two star systems of mass $M_1 = 2 \times 10^{30}$ kg and $M_2 = 3 \times 10^{30}$ kg are orbiting in their own orbitals as shown in figure to the right and if they experience 6×10^{25} N force during the generation of gravitational wave, Find out the separation distance between them?



5. **Keplers Law (20%)**

(a) What are the three Kepler's laws?

(b) If the radius of a planet orbit is r , the mass of the planet is M_p , moving with a velocity V in its orbit, prove the second Kepler's law.

6. **Sound Wave (20%)**: Refer to the following figure, suppose a firework charge is detonated at the top of Taipei-101 as show in figure. Due to the explosion if the acoustic pressure is reached the maximum of $\Delta P_{\max} = 20$ Pa at the distance of 1st person $d_1 = 1 \times 10^3$ m from the explosion, what sound level will be experienced by the 2nd person at a distance of $d_2 = 4 \times 10^3$ m? Let the speed of sound is constant at 332 m/s throughout the atmosphere, the air absorption rate of sound energy = 10 dB/km, the density of air $\rho = 1.2$ kg/m³ and $I_0 = 10^{-12}$ W/m².

