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General Physics-II, Quiz 9
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St. ID:__________________ Name:________________

Chapter 26-27, Serway; ABSOLUTELY NO CHEATING!
Please write down the answers on the blank space or on the back of this paper. Answer should be in english. [ ] indicates the question points.

1. Two capacitors, C_1 = 5.00 mF and C_2 = 12.0 mF, are connected in parallel, and the resulting combination is connected to a 9.00 V battery. Find (a) the equivalent capacitance of the combination and the potential difference across each capacitor. (b) What will be the results if they (C_1 &C_2) are connected in series? [25+25=50]

Solution:

(a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$$C_{\text{eq}} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = 17.0\mu F$$

The potential difference across each branch is the same and equal to the voltage of the battery.

$$\Delta V = 9.0V$$

(b) In series capacitors add as

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00\ \mu\text{F}} + \frac{1}{12.0\ \mu\text{F}}$$

$$C_{\text{eq}} = 3.53\mu F$$

The charge on the equivalent capacitor is

$$Q_{\text{eq}} = C_{\text{eq}}\Delta V = (3.53\ \mu\text{F})(9.00\ \text{V}) = 31.8\ \mu\text{C}$$

Each of the series capacitors has this same charge on it.

So $$Q_1 = Q_2 = 31.8\mu C$$

The potential difference across each is

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{31.8\ \mu\text{C}}{5.00\ \mu\text{F}} = 6.35V$$

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{31.8\ \mu\text{C}}{12.0\ \mu\text{F}} = 2.65V$$

2. If you want to fabricate a uniform wire from 1.00 g of copper and the wire is to have a resistance of R = 0.500 Ω and all the copper is to be used, what must be (a) the length and (b) the diameter of this wire ? (c) If the magnitude of the drift velocity of free electrons in this copper wire is 7.84 x 10^{-4} m/s, what is the electric field in the conductor? [20+10+20]

Solution:

(a) The total mass can be written as $m = \rho_m V = \rho_m A\ell \quad \rightarrow \quad A = \frac{m}{\rho_m \ell}$, where

$\rho_m \equiv$ mass density.

Taking $\rho \equiv$ resistivity, $R = \frac{\rho\ell}{A} = \frac{\rho\ell}{m/\rho_m\ell} = \frac{\rho\rho_m\ell^2}{m}$.

Thus,

$$l = \sqrt{\frac{mR}{\rho\rho_m}} = \sqrt{\frac{(1.00\times10^{-3}\text{ kg})(0.500\ \Omega)}{(1.70\times10^{-8}\ \Omega\cdot\text{m})(8.92\times10^{3}\text{ kg/m}^3)}}$$
$$= 1.82m$$

(b) The total volume, $V = \frac{m}{\rho_m}$, or $\pi r^2 \ell = \frac{m}{\rho_m}$ for a wire

Thus,

$$r = \sqrt{\frac{m}{\pi\rho_m\ell}} = \sqrt{\frac{1.00\times10^{-3}\text{ kg}}{\pi(8.92\times10^{3}\text{ kg/m}^3)(1.82\text{ m})}} = 1.40\times10^{-4}\text{ m}$$

The diameter is twice this distance: diameter $= 2.8\times10^{-4}m$

(c) The resistivity and drift velocity is related to the electric field within the copper wire

$$\rho = \frac{m}{ne^2\tau} \rightarrow \tau = \frac{m}{\rho ne^2}$$

and

$$v_d = \frac{eE}{m}\tau = \frac{eE}{m}\frac{m}{\rho ne^2} = \frac{E}{\rho ne}$$
$$\therefore E = \rho nev_d$$

Where n is the electron density and can be found by

$$n = \frac{N_A\rho_{Cu}}{M} = \frac{(6.02\times10^{23}\text{ mol}^{-1})(8\ 920\text{ kg/m}^3)}{0.063\ 5\text{ kg/mol}} = 8.46\times10^{28}\text{ m}^{-3}$$

The electric field is then

$$E = \rho nev_d$$
$$E = (1.7\times10^{-8}\,\Omega\cdot\text{m})(8.46\times10^{28}\text{ m}^{-3})$$
$$\times(1.60\times10^{-19}\text{ C})(7.84\times10^{-4}\text{ m/s})$$
$$= 0.18\text{V/m}$$