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General Physics-I, Quiz-2
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St. ID:________________________, Name:_________________

Chapter -10-11, Serway; ABSOLUTELY NO CHEATING!
Please write down the answers on the blank space or on the back of this paper. Answer should be in english. [ ] indicates the question points.

Q1. (a) Drive the equation of kinetic energy for an object of mass M while it is in angular motion. (b) Suppose you have an analogue watch with 2 cm long minute hand and 1 cm long hour hand. If their mass is 5 mg and 2 mg respectively, calculate the total kinetic energy of them with respect to the rotational axis. [20+30=50 %]

Solution: (a) In general case, kinetic energy of an object in linear momentum is

$k=\frac{1}{2}MV^2 ----------(i)$, using the definition of angular velocity we can write

$\omega=\frac{V}{r}\Rightarrow V=\omega r --------(ii)$, So from equation (i) and (ii) , we get

$k=\frac{1}{2}MV^2=\frac{1}{2}m(\omega r)^2=\frac{1}{2}Mr^2\omega^2=\frac{1}{2}I\omega^2$, where I = moment of inertia

(b) The moment of inertia of a thin rod about an axis through one end is $I=\frac{1}{3}ML^2$. The total

rotational kinetic energy is given as

$$K_R=\frac{1}{2}I_h\omega_h^2+\frac{1}{2}I_m\omega_m^2$$

With

$$I_h=\frac{M_hL_h^2}{3}=\frac{2\times10^{-3}\,\text{kg}\left(0.01\,\text{m}\right)^2}{3}=6.6\times10^{-8}\,\text{kg}\cdot\text{m}^2$$

and $$I_m=\frac{M_mL_m^2}{3}=\frac{5\times10^{-3}\,\text{kg}\left(0.02\,\text{m}\right)^2}{3}=6.6\times10^{-7}\,\text{kg}\cdot\text{m}^2$$

If a thin rod of mass M rotates with an axis through it one end

$I=\int_0^L r^2dm,\ dm=\lambda dx=\frac{M}{L}dx$

(See lecture P10-5)

$I=\int_0^L x^2\frac{M}{L}dx=\frac{M}{3L}[x^3]_0^L$

$I=\frac{M}{3L}L^3=\frac{1}{3}ML^2$

Now the rotational velocity for hour and second hands is,

$$\omega_h=\frac{2\pi\,\text{rad}}{12\,\text{h}}\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)=1.45\times10^{-4}\ \text{rad/s}$$

And $\omega_m = \dfrac{2\pi\,\text{rad}}{60\,m}\left(\dfrac{1\,\text{h}}{60\,\text{s}}\right) = 1.75\times10^{-3}\ \text{rad/s}$

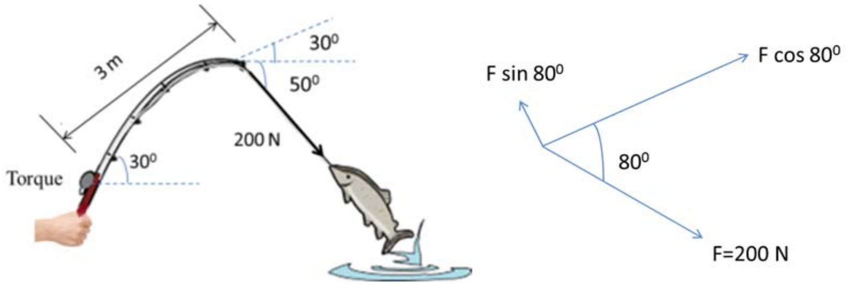
Therefore, total rotational kinetic energy is

$$K_R = \frac{1}{2}\left(6.6\times10^{-8}\ \text{kg}\cdot\text{m}^2\right)\left(1.45\times10^{-4}\,\text{rad/s}\right)^2 + \frac{1}{2}\left(6.6\times10^{-7}\,\text{kg}\cdot\text{m}^2\right)\left(1.75\times10^{-3}\,\text{rad/s}\right)^2 = 14.07\times10^{-16}\,\text{J}$$

Q2. If the Dong Hwa University Lake is open for fishing and you drop a fishing pole with angle of 30° according to horizontal axis as shown in figure below, what is the torque exerted by the fish about an axis perpendicular to the page and passing through your hand if the fish pulls on the fishing line with a force **F**= 200 N at an angle 50° below the horizontal? The force is applied at a point 3m from your hands. [10+ 40 = 50%]

Solution:

The force 200-N is exposed with components perpendicular to and parallel to the rod, so the components could be written as



F_{parallel} = (200 N) cos (50+30) ° = 34.7 N

And

$F_{\text{perpendicular}}$ = (200 N) sin (50+30) ° = 196.9 ~200N

The torque of F_{parallel} is zero since its line of action passes through the pivot point. The torque of $F_{\text{perpendicular}}$ is τ = (200N) x 3m = 600 N-m (Clockwise direction)

Q3. Is the angular momentum a vector quantity, explain why? If the second hand of your watch is designed with a bob of mass 5g at the end point as shown in figure, calculate the angular momentum of the bob. Let the radius covered by the second hand is 2 cm and takes 60s to travel 1 cycle. [10+20= 30%]

Solution:

Part 1: Yes, angular momentum is a vector quantity. Angular momentum is created while an object is in angular motion/ circular motion with respected to an axis, so the velocity of the object for a point is always directional ($\vec{v}$) / in tangential direction and the force created by the object is towards the center which is perpendicular direction ($\vec{r}$) to the velocity (through the radius of circle). So the angular momentum is created in a direction which is perpendicular to both ($\vec{v}$) x ($\vec{r}$). Therefore, by definition Angular momentum is always a vector cross product, $\vec{L} = \vec{r} \times m\vec{v} = \vec{r} \times \vec{p}$.

Part 2: For a cycle the bob passes the distance = 2πr, the velocity of the bob is V = 2πr/t

So the angular velocity is $\omega = \frac{v}{r} = \frac{2\pi r}{rt} = \frac{2\pi}{t} = \frac{2?\ \ .14 \text{ radian}}{60\text{s}} = 0.104\text{rad/s}$

The moment of inertia of the bob is $I = mr^2 = 5g \times (2cm)^2 = (5\times10^{-3})\times(2\times10^{-2}) = 1\times10^{-4} kg - m^2$

(Here the moment of inertia is calculated only for the bob, see the problem- P11.11)

Therefore, the angular momentum is $L = I\omega = 1\times10^{-4} kg - m^2 \times (0.104) rad/s = 10.6\times10^{-6} kg - m^2/s$