

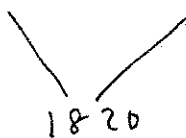
# Chap 23 Electric fields

## Chapter 22. Electric Charges

23-1

Electromagnetism  $\left\{ \begin{array}{l} \text{Electric (Electricity)} \\ \text{magnetic (Magnetism)} \end{array} \right.$   
 $E + M$

Electricity                      magnetism



Oersted : moving current create magnetic field



Michael Faraday

James C. Maxwell (put in mathematic form)



Maxwell's Equations (Table 32-1, P. 803)

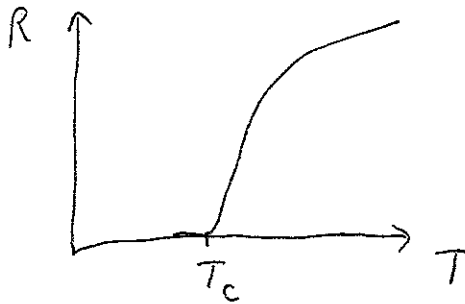
1. Electric Charges — intrinsic characteristic of the fundamental Charges
  - Static cling
  - Spark between fingers and door knobs in a dry day
  - lightning in the sky
2. positive Charges — assigned arbitrarily by B. Franklin  
negative charges —  
electric neutral — balance between <sup>positive</sup> positive + negative charges  
no net charges.
3. Interaction of charged objects through electric force.
  - Charges with the same electrical sign repel each other,  
charges with the opposite sign attract each other.
  - Coulomb's law, (electrostatic force, electric force)

4. Conductor, - negative charge move freely

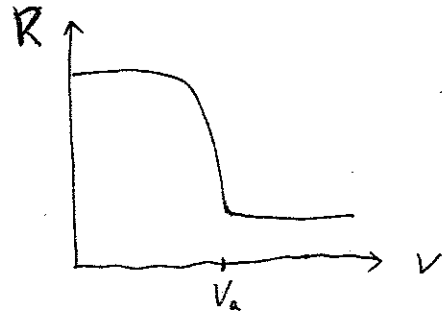
insulator - charges don't move.

Semi conductor - at some apply voltage charge will move  
Si, Ge, ...

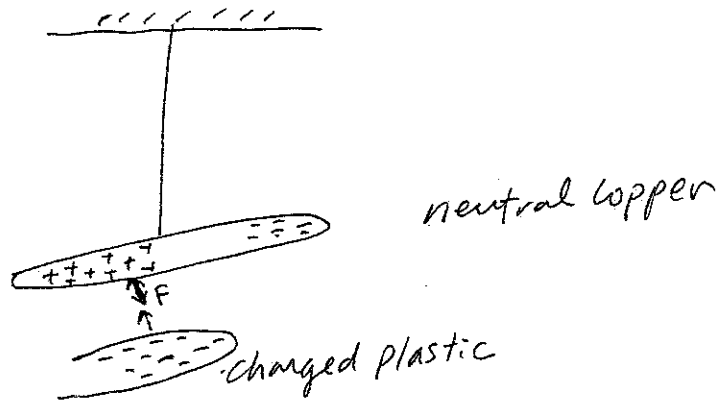
Super conductor -



Super conductor

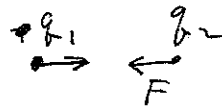


Semi conductor



5. Coulomb's law - Charles Augustin Coulomb's

$$|F| = k \frac{|q_1 q_2|}{r^2}$$



$$\rightarrow F = k \frac{q_1 q_2}{r^2}$$

Compared with Newton's force

$$F = -G \frac{M_1 M_2}{R^2}$$

$$\rightarrow F = \pm k \frac{|q_1 q_2|}{r^2}$$

"+" repel

"-" attract

Compare All forces

$k$  = electrostatic constant

$$= \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \frac{\text{m}^2}{\text{C}^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 = \text{permittivity constant.}$$

$q$ : in Coulomb, SI unit.

Q: SI unit, Coulomb

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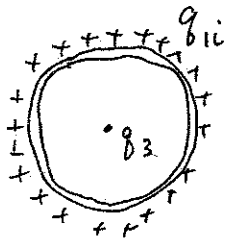
1 Coulomb = 1A  → 1A in one second.

$$dq = i dt$$

$$\rightarrow F_{12} = k \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Electric force follow the same vector sum as that of gravitational force, ie,  $\vec{F}_1 = F_{12} + F_{13} + F_{14} + \dots$ , and the force on one particle is the sum of all other particles force on this one.

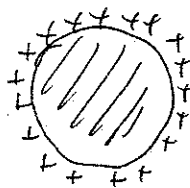
6. Shell:



$q_2$  the force on  $q_2$  is the sum of all  $q_i$ 's as if they were centered on the center of the shell

But  $q_3$  will exert no force due to all  $q_i$ 's in the shell.

Sphere (conductor)

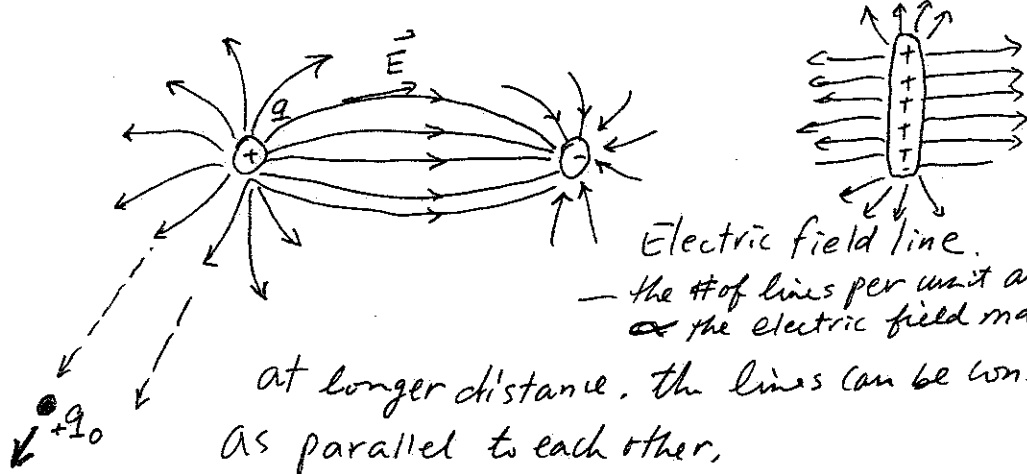


Charges will spread over the surface of a sphere uniformly, because charges will repel one another

7 charge is "Quantized" — ie,  $q = ne$ ,  $e =$  charge of an electron  
 $e$  is the elementary charge.  
 $= 1.6 \times 10^{-19}$  Coulombs

8 charge is conserved

Electric field — Set up by charge, within it, another charge is subject to force according to Coulomb's law



Electric field line.  
— the # of lines per unit area  $\propto$  the electric field magnitude.

at longer distance, the lines can be considered as parallel to each other.

If we place a test charge  $q_0$ .

$$F = k \frac{qq_0}{r^2}$$

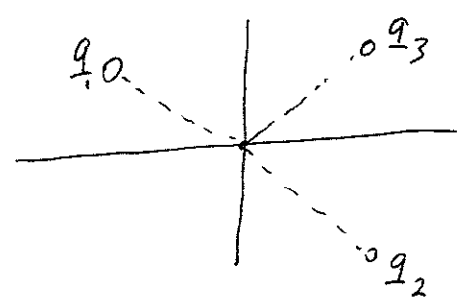
$$\vec{E} \equiv \frac{\vec{F}}{q_0} = k \frac{q}{r^2}, \text{ Electric field. } \left[ \frac{N}{C} \right]$$

- ① Electric field lines are drawn from positive charge to negative charge
- ② The # density of lines is proportional to the magnitude of the electric field.

1. Electric field — discrete charge

① point charge :  $\vec{E} = \frac{F}{q} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

② multiple point charges



$\vec{E}_0$  at the origin  
 $\vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

# Charge distribution

2335

## ④ Ring of charge distribution.

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(z^2 + R^2)}$$

$$dq = \lambda ds$$

$$\vec{E} = \int d\vec{E} \cos\theta = \int \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(z^2 + R^2)} \cdot \frac{z}{\sqrt{z^2 + R^2}} ds$$

$$= \frac{\lambda}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}} \cdot \int_0^{2\pi R} ds$$

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{R^2 + z^2}}$$

$$= \frac{2\pi R \lambda}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}} = \frac{1}{4\pi\epsilon_0} \frac{qz}{(z^2 + R^2)^{3/2}}$$

if  $z \gg R$ ,  $z^2 + R^2 \approx z^2$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{z^2}$$

At a large distance, a ring distribution can be approximated as the point charge.

Note: 1) Always check the answer at extreme case

2) Do sample problem 23-5 for a section of ring distribution

## ⑤ Charged disk (charge on its upper surface)

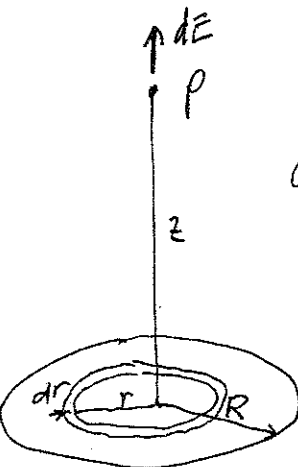
$$dq = \sigma dA = \sigma (2\pi r dr)$$

$$d\vec{E} = \frac{z \sigma 2\pi r dr}{4\pi\epsilon_0 (z^2 + r^2)^{3/2}}$$

$$= \frac{\sigma z}{4\epsilon_0} \frac{2r dr}{(z^2 + r^2)^{3/2}}$$

$$E = \int_{r=0}^{r=R} d\vec{E} = \frac{\sigma z}{4\epsilon_0} \int_0^R \frac{2r dr}{(z^2 + r^2)^{3/2}}$$

$$= \frac{\sigma z}{4\epsilon_0} \left[ \frac{(z^2 + r^2)^{-1/2}}{-1} \right]_0^R = \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

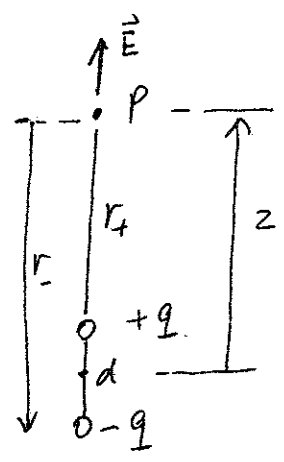


$\sigma$  = Charge density

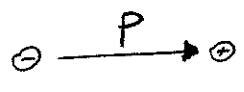
③ dipole —  $\begin{matrix} +q & & -q \\ & \text{---} & \\ & d & \\ & & 0 \end{matrix}$

two charges separated by a distance, have opposite sign are called electric dipole

$$\begin{aligned} \vec{E} &= \vec{E}_{(+)} + \vec{E}_{(-)} = \text{electric field @ pt. P.} \\ &= \frac{1}{4\pi\epsilon_0} \left[ \frac{q}{r_+^2} - \frac{q}{r_-^2} \right] \\ &= \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{[z - \frac{1}{2}d]^2} - \frac{1}{[z + \frac{1}{2}d]^2} \right] \\ &= \frac{q}{4\pi\epsilon_0 z^2} \left[ \left(1 - \frac{d}{2z}\right)^{-2} - \left(1 + \frac{d}{2z}\right)^{-2} \right], \quad z \gg d, \quad \frac{d}{2z} \ll 1 \\ &= \frac{q}{4\pi\epsilon_0 z^2} \left[ \left(1 + \frac{2d}{2z} + \dots\right) - \left(1 - \frac{2d}{2z} + \dots\right) \right] \\ &\approx \frac{q}{4\pi\epsilon_0 z^2} \frac{2d}{z} \\ &= \frac{1}{2\pi\epsilon_0} \frac{qd}{z^3} \\ &= \frac{1}{2\pi\epsilon_0} \frac{P}{z^3} \end{aligned}$$



$P \equiv qd = \text{electric dipole moment.}$



in general

- i)  $E \propto \frac{1}{r^3}$  for all dipole at all distance
- ii) The direction of the dipole is taken from the negative to the positive charge  $\leftarrow \ominus \rightarrow \oplus \leftarrow$
- iii) At a distance,  $\vec{E}$  is the same direction of  $\vec{P}$
- iv)  $E \propto \frac{1}{r^2}$  for a point charge.
- v)  $H_2O$ .  $p = 6.2 \times 10^{-30}$  C.m



for a charged disk  $E = \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$

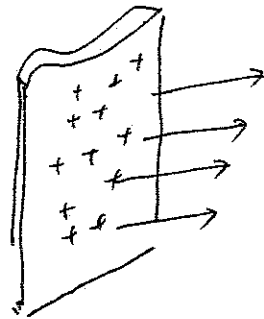
⑥ Infinite large sheet

from E for the disk,  $E = \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$

if  $R \gg z$ ,  $R \rightarrow \infty$ ,  $\frac{z}{\sqrt{z^2 + R^2}} \approx 0$

$E \rightarrow \frac{\sigma}{2\epsilon_0}$  — Electric field for a large sheet.

$E \propto \sigma$ , from different pt of view.



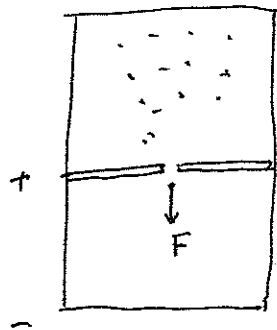
$\vec{E}$  must be uniform and  $\propto \sigma$ , the surface charge density.

2. Electric force — for a pt charge in an electric field

$$\vec{F} = q\vec{E}$$

Note: pt charge doesn't mean it is Unit Charge.

1) Millikan Oil drop exp.

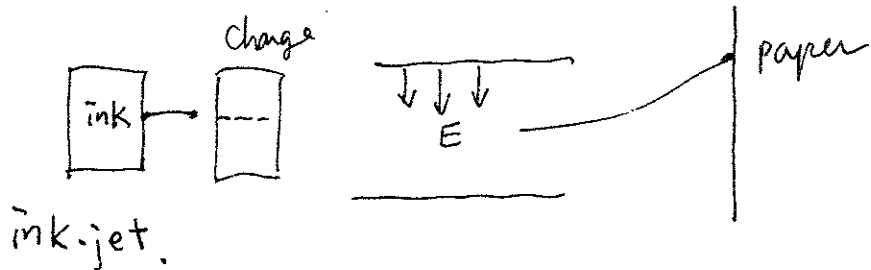


$$F = qE$$

$$q = ne$$

$$e = 1.6 \times 10^{-19} \text{ Coulomb.}$$

2)

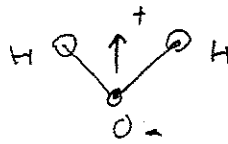


ink-jet.

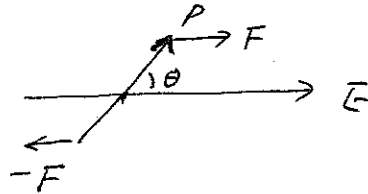
### 3. dipole in an electric field.

23-8

H<sub>2</sub>O



H<sub>2</sub>O has permanent dipole.



Force due to the interaction of Electric field and charge in a dipole, the <sup>net</sup> force is zero. but exert a net torque about its center of mass.

$$\tau = F \frac{d}{2} \sin \theta + F \frac{d}{2} \sin \theta = Fd \sin \theta$$

$$= qEd \sin \theta$$

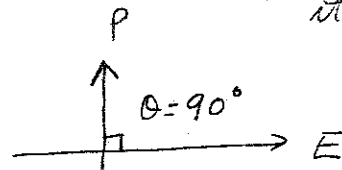
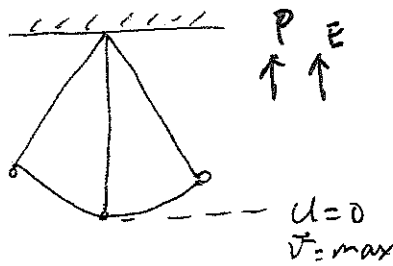
$$\tau = pE \sin \theta = \vec{p} \times \vec{E} = -pE \sin \theta \quad (\text{negative torque})$$

→ this give rise to the rotation of the molecules.

But the potential energy is minimum when the dipole is lined up along the external field. i.e.  $\tau = p \times E = 0$ ,  $U = 0$

Compared to a pendulum, at this point, the dipole is at its equilibrium position

it has the least potential



$U = \text{max}$  defined potential =  $\tau = \text{max}$ . ~~define the potential.~~

$$\therefore U = -W = - \int_{90^\circ}^{\theta} \tau d\theta = + \int_{90^\circ}^{\theta} pE \sin \theta d\theta \quad \text{— potential at any other angle}$$

$$\therefore U = -p \cdot E \cos \theta = -p \cdot E \quad \downarrow$$

### 4. Micro waver.

Note:  $\tau = -pE \sin \theta$   
negative torque

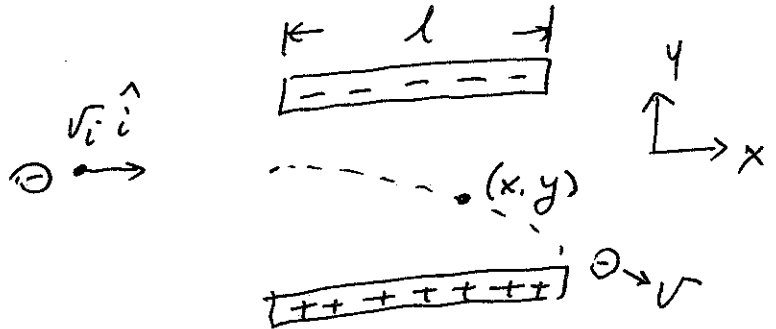
as in the figure, it gives rise to a clockwise rotation



23.7

Motion of a charge particle in a Uniform electric field

$$F_e = qE = ma \rightarrow a_y = \frac{qE}{m}$$



$$v_x = v_i = \text{constant}$$

$$v_y = a_y t = -\frac{eE}{m_e} t$$

$$x_f = v_i t$$

$$y_f = \frac{1}{2} a_y t^2 = -\frac{1}{2} \frac{eE}{m_e} t^2$$

### 23.5. Electric field of a Continuous Charge distribution

$$\vec{\Delta E} = k_e \frac{\Delta q}{r^2} \hat{r}$$

$$\therefore \vec{E} \sim k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

if the charge is continuous distributed

$$\vec{E} = k_e \lim_{\Delta q_i \rightarrow 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

$$= k_e \int \frac{dq}{r^2} \hat{r}$$

