Chap 17 Sound waves

Speed of Sound wave $v = \sqrt{\frac{B}{\rho}}$ $B \equiv$ Bulk Module, $\rho =$ density

$v = \sqrt{\frac{T}{\mu}}$ — Speed in a String (16.8)

$\rightarrow$ $v_{mechanics} = \sqrt{\frac{\text{elastic property}}{\text{Inertial property}}}$ check Table 17.1 Page 514

Speed of sound also depends on the temperature

$$v = 331 (m/s) \sqrt{1 + \frac{T_c}{273°c}}$$

17.2. periodic Sound Waves

$$s(x.t) = s_{max} \cos(kx - \omega t)$$

s_{max} = maximum position of the element relative to the equilibrium.

$$\Delta p = \Delta P_{max} \sin(kx - \omega t)$$

$$= \rho v \omega s_{max} \sin(kx - \omega t)$$

<u>Derivation</u> $\Delta p = -B \frac{\Delta V}{V_i}$ (12.8)

$$= -B \frac{A \Delta s}{A \Delta x} = -B \frac{\Delta s}{\Delta x} = -B \frac{\partial s}{\partial x}$$

$$\therefore \Delta p = -B \frac{\partial}{\partial x}\left[s_{max} \cos(kx - \omega t) \right] = B s_{max} k \sin(kx - \omega t)$$

But $B = \rho v^2$, $k = \frac{\omega}{v}$

$$\therefore \Delta p = \rho v^2 s_{max} k \sin(kx - \omega t) = \rho v^2 \frac{\omega}{v} s_{max} \sin(kx - \omega t)$$

$$\therefore \Delta p = \rho v \omega s_{max} (kx - \omega t)$$

17.3 Intensity of the periodic Sound wave.

$$v(x,t) = \frac{\partial}{\partial t} s(x,t) = \frac{\partial}{\partial t}\left[s_{max} \cos(kx - \omega t)\right]$$

$$= -\omega s_{max} \sin(kx - \omega t)$$

at $t=0$. the kinetic energy is

$$\Delta K = \frac{1}{2}\Delta m v^2 = \frac{1}{2}\Delta m \left[-\omega s_{max} \sin(kx - \underbrace{\omega t}_{t=0})\right]^2$$

$$= \frac{1}{2}\rho A \Delta x \,(\omega s_{max})^2 \sin^2(kx)$$

if $\Delta x \to$ small $\to dx$

$$dK = \frac{1}{2}\rho A\, dx\, \omega^2 s_{max}^2 \sin^2(kx)$$

$$K_\lambda = \int dK = \int_0^\lambda \frac{1}{2}\rho A \omega^2 s_{max}^2 \sin^2(kx)\, dx$$

$$= \frac{1}{2}\rho A \omega^2 s_{max}^2 \int_0^\lambda \sin^2(kx)\, dx$$

$$= \frac{1}{4}\rho A \omega^2 s_{max}^2 \lambda$$

Total mechanical energy E_λ

$$E_\lambda = K_\lambda + U_\lambda = \frac{1}{4}\rho A \omega^2 s_{max}^2 \lambda + \frac{1}{4}\rho A \omega^2 s_{max}^2 \lambda$$

$$= \frac{1}{2}\rho A (\omega s_{max})^2 \lambda$$

$P = \frac{\Delta E}{\Delta t}$ = rate of energy transfer

$$= \frac{\frac{1}{2}\rho A (\omega s_{max})^2 \lambda}{T} = \frac{1}{2}\rho A (\omega s_{max})^2 \left(\frac{\lambda}{T}\right)$$

$$= \frac{1}{2}\rho A v (\omega s_{max})^2$$

$$I \equiv \frac{P}{A} = \frac{1}{2}\rho v (\omega s_{max})^2 = \frac{\Delta P_{max}^2}{2\rho v}$$

Now Consider a point Source

$$I = \frac{P_{Av}}{A} = \frac{P_{Av}}{4\pi r^2} \sim \frac{1}{r^2}$$

Decibels db β

$$\beta = 10 \log\left(\frac{I}{I_0}\right), \quad I_0 = \text{reference intensity}$$

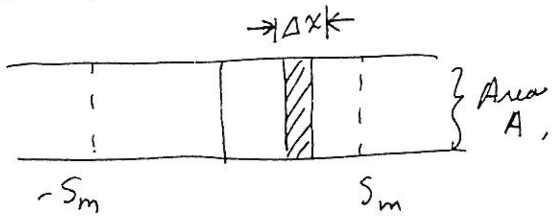
$= \text{threshold of hearing}$

$= dB \qquad = 1.0 \times 10^{-12} \frac{W}{m^2}$

$$I = \frac{\text{Watts}}{(m)^2}$$

17.4 Doppler Effect

Intensity again, consider the same Δx (dx) volume element of air,



Mass $dm = \rho A dx$

$$dE_K = \frac{1}{2} dm\, V_s^2 = \frac{1}{2} dm \left[\frac{\partial s}{\partial t}\right]^2 = \frac{1}{2} dm \left[\frac{\partial}{\partial t} S_m \cos(kx - \omega t)\right]^2$$

$$= \frac{1}{2} dm \left[\omega S_m \sin(kx - \omega t)\right]^2$$

$$= \frac{1}{2} (\rho A dx)\, \omega^2 S_m^2 \sin^2(kx - \omega t)$$

$$\frac{dE_K}{dt} = \frac{1}{2} \rho A v\, \omega^2 S_m^2 \sin^2(kx - \omega t) \equiv$$ Rate of change in kinetic energy.

$$\overline{\left(\frac{dE_K}{dt}\right)} = \frac{1}{2} \rho A v \omega^2 S_m^2\, \overline{\sin^2(kx - \omega t)}$$

$$= \frac{1}{4} \rho A v \omega^2 S_m^2 \Rightarrow \overline{\left(\frac{du}{dt}\right)} = \frac{1}{4} \rho A v \omega^2 S_m^2$$

$$\boxed{I = \frac{2\overline{\left(\frac{dE_K}{dt}\right)}}{A} = \frac{1}{2} \rho \cancel{A} v \omega^2 S_m^2}$$

Assume the rate change of potential energy is the same as that of kinetic energy.

4. Beats. — the difference between two combining frequencies, ~~that~~ these two fs are be very close

$$S_1 = S_m \cos(\omega_1 t), \quad S_2 = S_m \cos \omega_2 t$$

$$S = S_1 + S_2 = S_m \left[\cos \omega_1 t + \cos \omega_2 t\right]$$

$$= S_m \cdot 2 \cos \tfrac{1}{2}(\omega_1 - \omega_2) t \cdot \cos \tfrac{1}{2}(\omega_1 + \omega_2) t$$

$$= \left[2 S_m \cos \omega' t\right] \cos \omega t, \quad \omega' = \tfrac{1}{2}(\omega_1 - \omega_2)$$
$$\omega = \tfrac{1}{2}(\omega_1 + \omega_2)$$

$$S = \left[2 S_m \cos \omega' t\right] \cos(\omega t) \, , \quad \omega > \omega'$$

- treat this as a function of $\cos \omega t$ with $2 S_m \cos \omega' t$ Amplitude.
- $\cos \omega t$ function with $2 S_m \cos \omega' t$.
- Maximum amplitude occurs when $\cos \omega' t = \pm 1$

$\rightarrow \frac{1}{2}(\omega_1 - \omega_2) t = \pm 2\pi$

- during a repetition of $\cos \omega t$, maximum amplitude happens Twice, therefore

$\cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t = \pm 1$

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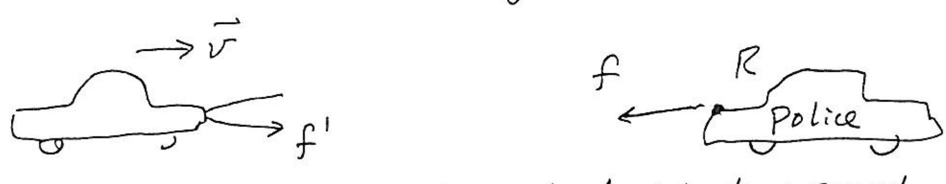
$$\omega_{beat} = 2\omega' = 2 \cdot \frac{1}{2}(\omega_1 - \omega_2)$$
$$= \omega_1 - \omega_2$$

But $\omega = 2\pi f$.

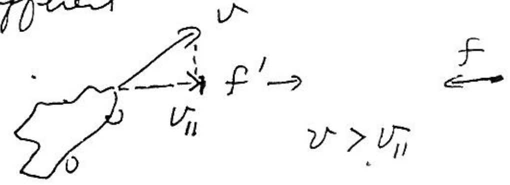
$$\rightarrow \boxed{f_{beat} = f_1 - f_2}$$

5. Doppler Effect

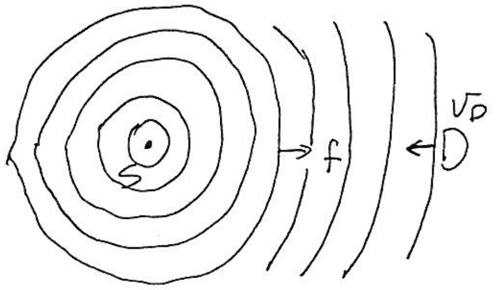
- The detected sound frequency change due to relative motion of the source and detector.
- True for wave, (EM + Sound).
- Radar detector of police use Doppler Effect to detect the speed of car (Using Microwave)



Due to misalignment, the actual detected speed of the car may be different



5-1. Detector Moving ; Source Stationary



Since the detector is moving, for a given interval of time Δt, detector will intercept more wave fronts.

$$f' > f$$

In time t, the wave front move to the right vt.

① Source → vt distance
detector ← $v_D t$
distance $d_{s,d}$

$$\lambda = \frac{v}{f}$$

$$\underset{S}{\bullet}\!\xrightarrow{\;vt\;}\;\xleftarrow{\;V_D t\;}\!\underset{D}{\bullet}$$

Detect frequency $f' = \frac{(vt + v_D t)/\lambda}{t} = \frac{v + v_D}{\lambda}$

$$= \frac{(v + v_D) f}{v}$$

$$\boxed{f' = f\left(\frac{v + v_D}{v}\right)}$$ – detector towards Source

② Source → vt distance
detector → $v_D t$ distance

$$d_{s,d} = vt - v_D t$$

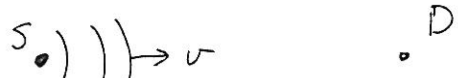
$$\boxed{f' = f\left(\frac{v - v_D}{v}\right)}$$ – detector away from Source

$$\Rightarrow \quad f' = f\left(\frac{v \pm v_D}{v}\right)$$

5-2. Source moving; detector stationary

— When sources are moving, the wave length changes.

① when S moves toward D



wave front W_1, vT during a period time T
Source move $V_s T$ " T, and emits a 2nd wave front W_2

— the distance between the W_1 and W_2 is the detected wave length of λ'

$$\lambda' = v\cancel{T} - V_s T$$

$$f' = \frac{v}{\lambda'} = \frac{v}{vT - V_s T} = \frac{v}{\frac{v}{f} - \frac{V_s}{f}} = f\,\frac{v}{v - V_s}$$

② when S moves away from D

$$f' = \frac{v}{\lambda} = \frac{v}{vT + V_s T} = \frac{v}{\frac{v}{f} + \frac{V_s}{f}} = f\,\frac{v}{v + V_s}$$

$$\Rightarrow \boxed{f' = f\,\frac{v}{v \mp V_s}}$$

General form

$$\boxed{f' = f\,\frac{v \pm V_D}{v \mp V_s}}$$

— for both Source and detector are moving
$v < V_D$
$< V_s$

$$f' = f\,(v \pm V_D)(v \mp V_s)^{-1}$$

$$= f\cdot\left(1 \pm \frac{V_D}{v}\right)\left(1 \mp \frac{V_s}{v}\right)^{-1}$$

$$= f\left[1 \pm \frac{V_D}{v} + \cdots\right]\left[1 \pm \frac{V_s}{v} + \cdots\right] \qquad u = |V_s \pm V_D|$$

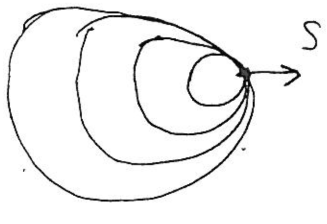
$$= f\left(1 \pm \frac{V_s}{v} \pm \frac{V_D}{v} + \frac{V_s V_D}{v^2} + \cdots\right)$$

$$= f\left(1 \pm \frac{V_s \pm V_D}{v} + \cdots\right). \Rightarrow \boxed{f' = f\left(1 \pm \frac{u}{v}\right)}$$

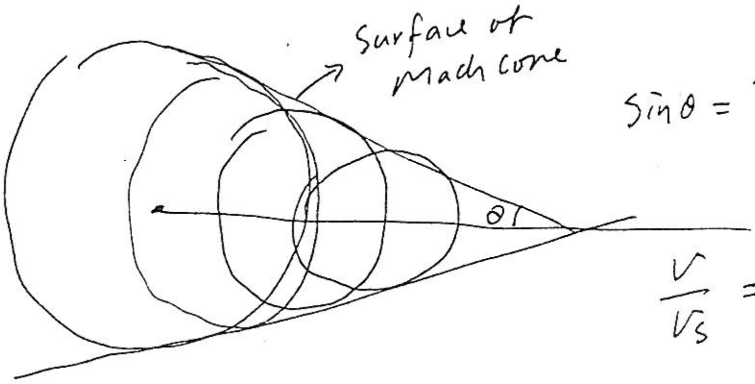
$$f' = f \cdot \frac{v \pm v_D}{v \mp v_S}$$

When $v = v_S$ towards the ~~source~~ detector, $f' \to \infty$

Shock wave appears
→ Sonic boom



$V_S = V$



$$\sin\theta = \frac{vt}{V_S t} = \frac{v}{V_S}$$

$\frac{V}{V_S}$ = Mach number

6. Doppler Effect of light.

$f' = f\left(1 \pm \frac{u}{c}\right)$ — for light wave; $u \ll c$

$$\lambda' = \lambda\left(1 \pm \frac{u}{c}\right)^{-1} \approx \lambda\left(1 \mp \frac{u}{c}\right)$$

or $\frac{\lambda' - \lambda}{\lambda} = \pm \frac{u}{c}$

or $\frac{\lambda' - \lambda}{\lambda} = \frac{\Delta\lambda}{\lambda} = \pm \frac{u}{c}$, $\Delta\lambda$ Doppler shift.

— measure the relative speed of a light source by measuring the Doppler shift.