

Chap 15 Oscillatory Motion

P15-1

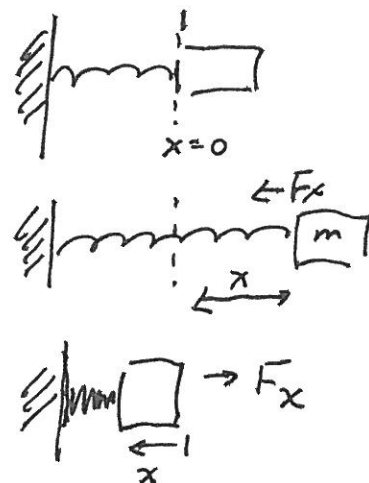
15.1 Motion of an object attached to a spring

$$F_s = -kx = ma_x$$

$$\therefore a_x = -\frac{k}{m}x$$

1) acceleration is proportional to \vec{x} and ~~object~~ opposite to the direction of displacement

2) Simple harmonic Motion



Mathematical

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\text{let } \frac{k}{m} = \omega^2$$

$$\frac{d^2x(t)}{dt^2} = -\omega^2 x(t) \quad \text{— 2nd order differential Equation}$$

$$x(t) = A \cos(\omega t + \phi)$$

$$\frac{dx}{dt} = A \frac{d}{dt} \cos(\omega t + \phi) = -\omega A \sin(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -\omega A \frac{d}{dt} \sin(\omega t + \phi) = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x$$

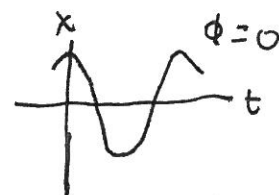
\therefore for a Simple Harmonic Oscillator

$$\ddot{x}(t) + \omega^2 x = 0$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A \cos(\omega t + \phi)$$

$\phi \equiv$ phase determined at $t=0$



$$x(t) = x(t+T)$$

$$[\omega(t+T) + \phi] - (\omega t + \phi) = 2\pi$$

$$\omega T = 2\pi \quad \text{or} \quad T = \frac{2\pi}{\omega} = 2\pi f$$

$$= 2\pi \sqrt{\frac{m}{k}}$$

$$x(t) = A \cos(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \quad , \quad v_{\max} = \pm \omega A$$

$$a = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi) \quad , \quad a_{\max} = \omega^2 A$$

① If we pick an initial condition

$$t=0 \quad x=A \quad \text{then} \quad x(0)=A$$

$$v(0)=0 \quad \text{initial condition}$$

$$\begin{cases} x(0) = A \cos \phi = A \\ v(0) = -\omega A \sin \phi = 0 \end{cases}$$

→ if $\phi=0$ $x = A \cos \omega t$ as solution

② If we define $t=0 @ x=0$

$$x(0) = A \cos \phi = 0 \quad \rightarrow \phi = \pm \frac{\pi}{2}$$

$$v(0) = -\omega A \sin \phi = v_i \quad \rightarrow A = \mp \frac{v_i}{\omega}$$

But initial velocity is positive
and amplitude must be positive

$$\rightarrow \phi = -\frac{\pi}{2}$$

$$\therefore x(t) = \frac{v_i}{\omega} \cos\left(\omega t - \frac{\pi}{2}\right)$$

do example problems

15.3 Energy of the simple harmonic oscillator

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi)$$

$$U = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

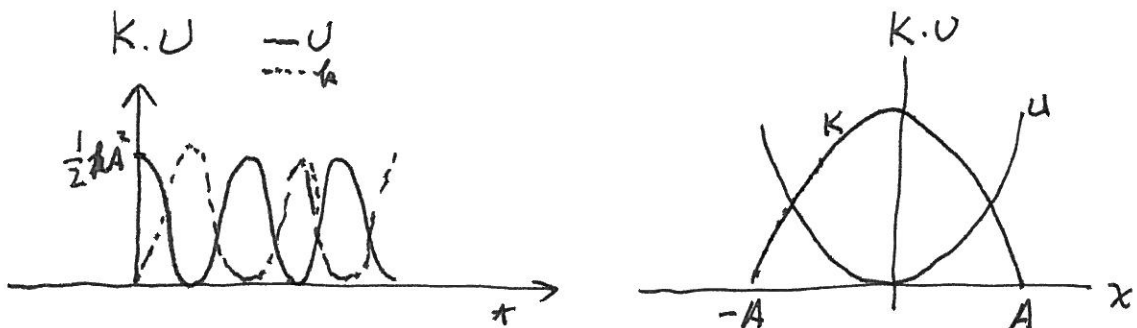
$$\bar{E} = K + U = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi) + \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} m \frac{k}{m} A^2 \sin^2(\omega t + \phi) + \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} k A^2$$

- total energy of a SHO is constant

- $\sim A^2$



Check the similarity of pendulum and SHO

Fig 15.11 (P463)

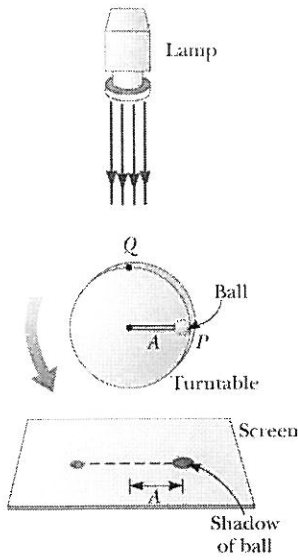
$$\bar{E} = K + U = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

$$\frac{1}{2} m v^2 = \frac{1}{2} k (A^2 - x^2)$$

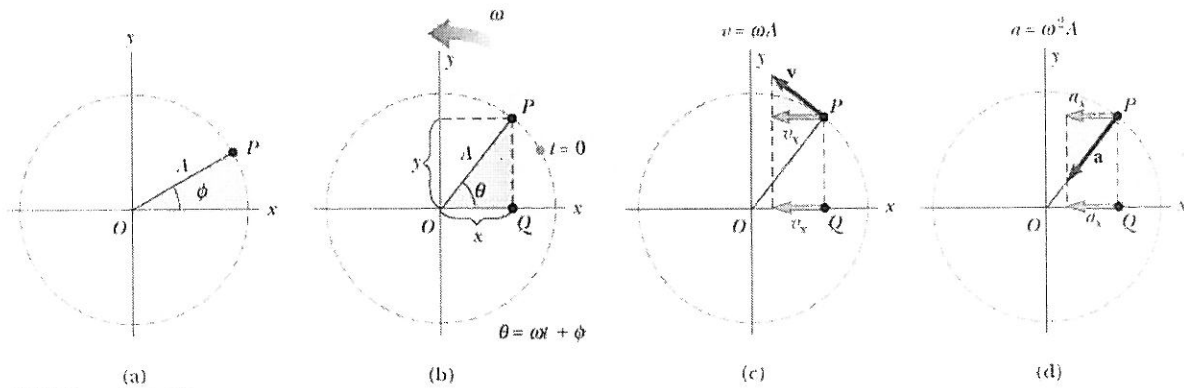
$$v = \pm \sqrt{\frac{k}{m} (A^2 - x^2)}$$

$$= \pm \omega \sqrt{A^2 - x^2}$$

Simple harmonic motion and Uniform Circular Motion

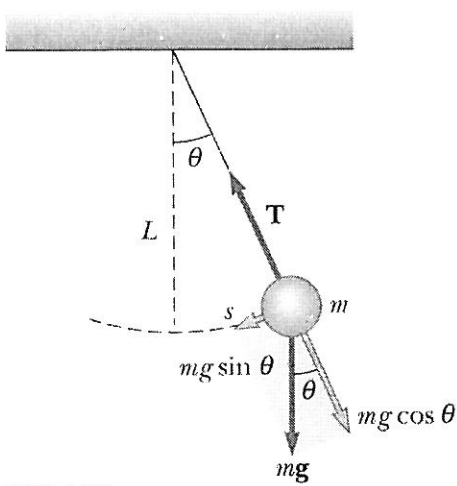


© 2004 Thomson/Brooks Cole



© 2004 Thomson/Brooks Cole

15.5 Pendulum



© 2004 Thomson/Brooks Cole

$$F_T = -mg \sin \theta = ma_T = m \frac{d^2 s}{dt^2}$$

$$\text{But } s = L\theta \quad \frac{d^2 s}{dt^2} = L \frac{d^2 \theta}{dt^2}$$

$$\therefore -mg \sin \theta = mL \frac{d^2 \theta}{dt^2}$$

$$\frac{d^2 \theta}{dt^2} = \left(-\frac{g}{L}\right) \sin \theta$$

$$\text{If } \theta \approx 0 \quad \sin \theta \approx \theta$$

$$\therefore \frac{d^2 \theta}{dt^2} = \left(-\frac{g}{L}\right) \theta$$

$$\omega = \sqrt{\frac{g}{L}} \quad \text{Same as that of a SHO}$$

$$T = 2\pi f = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$$

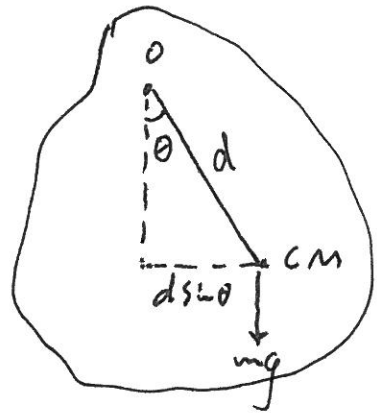
Physical pendulum

$$\tau = I \alpha$$

$$-mgd \sin \theta = I \frac{d^2 \theta}{dt^2}$$

$$\frac{d^2 \theta}{dt^2} = - \left(\frac{mgd}{I} \right) \theta = -\omega^2 \theta$$

$$\omega = \sqrt{\frac{mgd}{I}}$$



15.6 Damped Oscillator

$$\sum F_x = -kx - b \underbrace{v_x}_{\text{max}} = m \frac{d^2 x}{dt^2} \quad \text{--- } R$$

$$-kx - b \frac{dx}{dt} = m \frac{d^2 x}{dt^2}$$

damping term.

if b is small

$$x(t) = A e^{-\frac{b}{2m} t} \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

$$= \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

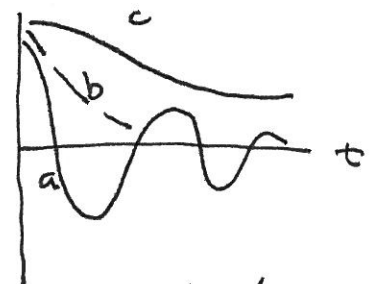
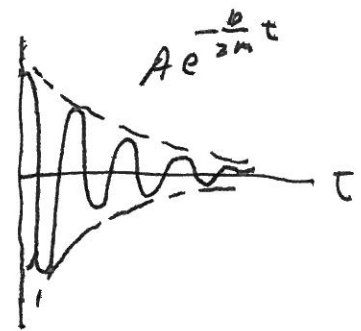
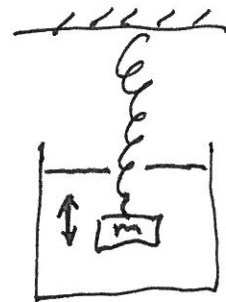
$$\omega_0 = \sqrt{\frac{k}{m}} \quad \text{--- natural frequency}$$

$$|R| = b v_y$$

a: $R_{\text{max}} < kA$ underdamped

b: $b_c = 2m\omega_0$ critically damped

c: $R_{\text{max}} > kA$ overdamped
 $= b v_{\text{max}}$



a: Underdamped

b: critically damped

c: overdamped oscillator

15.7 Forced Oscillation

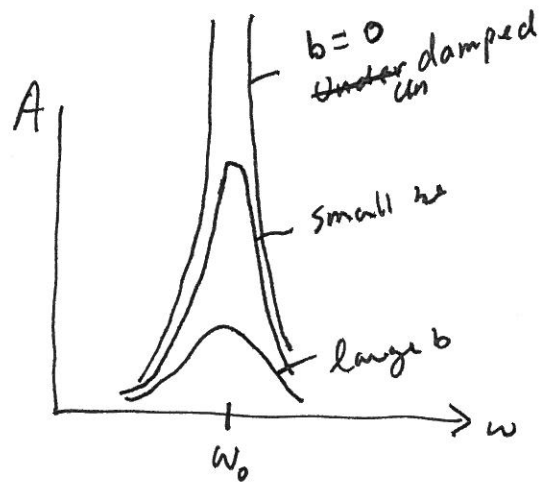
$$\Sigma F = ma \quad F_0 \sin \omega t - b \frac{dx}{dt} - kx = m \frac{d^2x}{dt^2}$$

↑
↓
 driving force damped term
 periodic

$$x = A \cos(\omega t + \phi)$$

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (\frac{b\omega}{m})^2}}$$

$\omega_0 =$ natural frequency when $b=0$



When the frequency of a driving force equals that of the natural frequency A increases for undamped oscillation