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## Quiz-Final-1

### 1. Solution:

$$(a) F = -kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

$$a_x = \frac{dV_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\therefore \frac{d^2x(t)}{dt^2} = -\frac{k}{m}x$$

$$\therefore \frac{d^2x(t)}{dt^2} + \frac{k}{m}x = 0$$

$$(b) F_T = -mg \sin \theta = mas = m \frac{d^2S}{dt^2}$$

$$\text{Where, } S = L\theta \quad \therefore \frac{d^2S}{dt^2} = L \frac{d^2\theta}{dt^2}$$

$$\Rightarrow -mg \sin \theta = mL \frac{d^2\theta}{dt^2}$$

$$\therefore \frac{d^2\theta}{dt^2} = -\left(\frac{g}{L}\right) \sin \theta$$

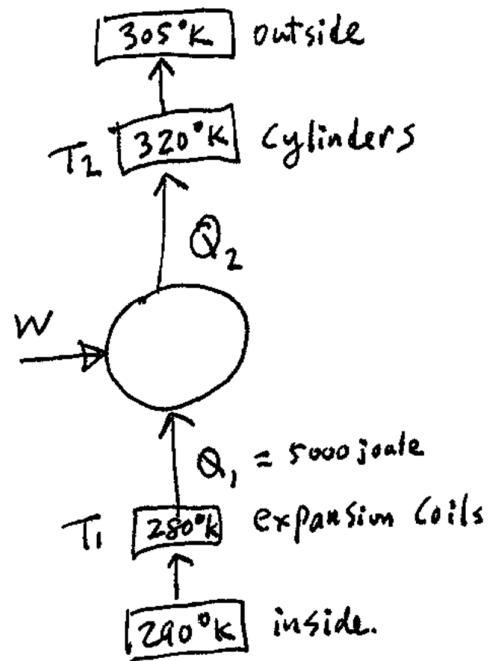
$$(c) \text{In this figure, } S=R\theta, \therefore \theta = \frac{S}{R} \Rightarrow \ddot{\theta} = \frac{\ddot{S}}{R}$$

$$\ddot{\theta} + \left(\frac{g}{L}\right)\theta = 0$$

$$\frac{\ddot{S}}{R} + \left(\frac{g}{L}\right)\frac{S}{R} = 0$$

$\therefore \ddot{S} + \left(\frac{g}{L}\right)S = 0$ , this is the same as in (a). So they both are Simple Harmonic Oscillators

2. Solution:



Schemically, all components are showing at the above.

(a) The efficiency  $\eta$  of a reversible heat engine

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1} \quad -\text{heating}$$

$$\eta = \frac{W}{Q_2} = \frac{Q_2 - Q_1}{Q_2} = \frac{T_2 - T_1}{T_2} \quad -\text{cooling}$$

$$\therefore \text{for Cooling } \eta = 1 - \frac{280}{320} = 0.125 = \frac{Q_2 - Q_1}{Q_2} = \frac{Q_2 - 5000\text{J}}{Q_2}$$

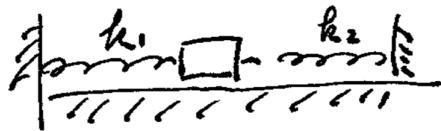
$$\therefore Q_2 = 5715\text{J} \Rightarrow W = \eta \cdot Q_2 = 0.125 \times 5715 \approx 715\text{J}$$

(b)

$$\Delta S = \frac{Q_2}{T_{\text{out}}} - \frac{Q_2}{T_{\text{cylinder}}} = Q_2 \left[ \frac{1}{305} - \frac{1}{320} \right]$$

$$= 0.88 \text{ J/K}$$

3. Solution:



$$\Delta x_1 = -\Delta x_2$$

If we generate a displacement  $\Delta x$ , then

$$\Delta x_1 = \Delta x$$

$$\Delta x_2 = -\Delta x$$

$$\begin{aligned} F &= -k_1 \Delta x_1 - (-k_2 \Delta x_2) = -k_1 \Delta x + k_2 (-\Delta x) \\ &= -(k_1 + k_2) \Delta x \\ &= -K' \Delta x, \quad K' = k_1 + k_2. \end{aligned}$$

$$V = \frac{1}{2\pi} \sqrt{\frac{K'}{m}} = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}}$$

4. Solution:

Adiabatic process for an ideal gas

$$Q = 0$$

$$\Delta E_{int} = Q + W = \overset{W}{\cancel{Q}} = n C_v dT = -P dV$$

$$\text{But } PV = n R T$$

$$\begin{aligned} P dV + V dP &= n R dT \\ &= -\frac{R}{C_v} P dV \end{aligned}$$

$$\begin{aligned} \frac{dV}{V} + \frac{dP}{P} &= -\left(\frac{C_p - C_v}{C_v}\right) \frac{dT}{T} \\ &\approx (1-\gamma) \frac{dT}{T} \end{aligned}$$

$$\frac{dP}{P} + \gamma \frac{dV}{V} = 0$$

$$\ln P + \gamma \ln V = \text{constant}$$

$$PV^\gamma = \text{constant}$$

5. Solution:



Let  $y_1 = A \sin(\omega x - \omega t)$  traveling to the right  
 $y_2 = A \sin(\omega x + \omega t)$  traveling to the left

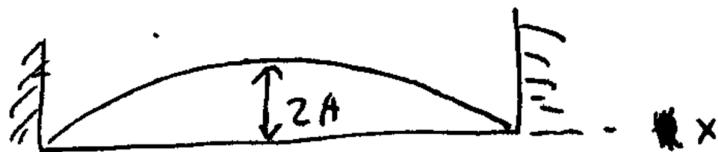
The resulting wave is

$$(a) \quad y = y_1 + y_2 = A \sin(\omega x - \omega t) + A \sin(\omega x + \omega t)$$
$$= \underline{2A \sin(\omega x)} \cos(\omega t)$$

(b) You have a new wave

$$y = \underline{2A} \sin(\omega x) \cos(\omega t)$$

Amplitude      Oscillation



This wave form oscillates as a function of  $\cos(\omega t)$ . It is not traveling anymore. It stands there, so it is called Standing Wave.

## 6. Solution :

(a) The absolute pressure is

$$P = P_0 + h\rho g, \text{ Where } P_0 \text{ is the air pressure and } h = 2.0 \text{ km}, \rho = 1000 \text{ kg/m}^3, g = 9.8 \text{ m/s}^2$$

$$\therefore P = 100000 \text{ Pa} + (2000 \text{ m} \times 1000 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2) = 1.97 \times 10^7 \text{ Pa}$$

(b) The gauge pressure is the difference in pressure between the water outside and the air inside the submarine, which we suppose is at 1.00 atmosphere.

$$\therefore P_{gauge} = P - P_0 = h\rho g = 1.96 \times 10^7 \text{ Pa}$$

The resultant inward force on the window is then

$$F = P_{gauge} A = 1.96 \times 10^7 \text{ Pa} \times \pi r^2 = 1.96 \times 10^7 \text{ Pa} \times 3.14 \times (2.0)^2 = 2.5 \times 10^8 \text{ N}$$

## 7. Solution:

(a) Kepler's third law of planetary motion is

$$T^2 = K_s a^3, \text{ where } T \text{ is period of planetary revolution, } a = \text{The semimajor axis length of orbit}$$

and  $K_s = \text{constant of proportionality (with respect to Sun)}$

(b) From figure we can find the major axis length of the orbit is

$$2a = 2.5 \therefore a = 1.25 \text{ AU}$$

and  $a+c=2.0 \therefore c=0.75 \text{ AU}$ , where  $c$  is the distance between focus and the center of orbital axis

$$\therefore \text{eccentricity } e = \frac{c}{a} = \frac{0.75}{1.25} = 0.60$$

(c) We know that  $T^2 = K_s a^3$

$$\begin{aligned} \text{So } T &= \sqrt{K_s a^3} = a \sqrt{K_s a} = 1.25 \text{ AU} \times \sqrt{2.97 \times 10^{-19} \times 1.25 \text{ AU}} \\ &= 1.25 \times 1.496 \times 10^{11} \times \sqrt{2.97 \times 10^{-19} \times 1.25 \times 1.496 \times 10^{11}} \\ &= 55165000 \text{ seconds} = \frac{55165000 \text{ days}}{3600 \times 24} = 638.5 \text{ days} = 1.78 \text{ years} \end{aligned}$$