

SN: _____, Name: _____

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are allowed.

The followings are some useful mathematics you may use without proof in answering your problems.

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots \quad \text{Time average } \overline{x(t)^n} = \langle x(t)^n \rangle = \frac{1}{T} \int_0^T x(t)^n dt$$

For a second order differential equation, $\frac{d^2x}{dt^2} + ax = 0$, the general solution of this equation is

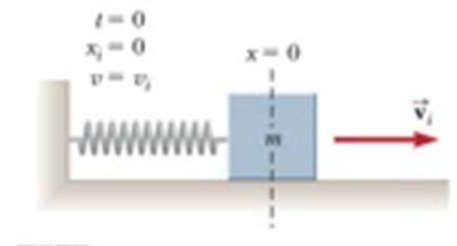
$$x(t) = x_0 \cos(at + \phi), \text{ where } x_0 \text{ is the maximum, and } \phi \text{ is the phase angle.}$$

$$N_A = 6 \times 10^{23}, R = \text{Gas constant} = 8.31 \text{ J/mole K, room temperature} = 300\text{K, } 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa.}$$

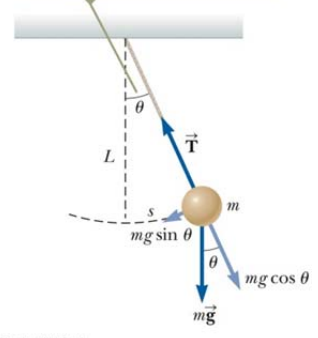
$$\overline{v_x} = \frac{v_{rms}}{\sqrt{3}} \text{ for ideal gas.}$$

Problems (7 Problems, total 130%)

- 1. Harmonic Oscillation (20%):** (a, b) Refer to the figure to the right, write down differential equations that can describe the motion for a mass m attached to a spring of force constant k and the same mass attached to a pendulum of length L . In both cases, use the given parameters. In the upper case, describe the motion in terms of its displacement x ; while in the lower case, describe its motion in terms of the angle θ . (c) If we treat the pendulum as a simple harmonic oscillator and look at only the horizontal displacement s at small angle, what will the equation look like?

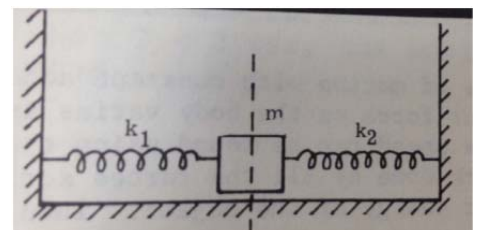


When θ is small, a simple pendulum's motion can be modeled as simple harmonic motion about the equilibrium position $\theta = 0$.



- 2. Entropy and thermodynamics (20%):** In an air conditioning process a room is kept at 290°K while the temperature outside is 305°K. The refrigerating machine has compression cylinders operating at 320°K (located outside) and expansion coil inside the house operating at 280°K. If the machine operates reversibly, how much work must be done for each transfer of 5000 joules of heat from the house? What is the entropy changes occur outside the house for this amount of refrigeration?

- 3. Spring system (20%):** For a spring system show to the right, calculating the frequency of oscillation of the configuration shown in the figure. All surfaces are



frictionless.

4. **Adiabatic Process (15%):** Prove that for an adiabatic expansion of an ideal gas, $PV^\gamma = \text{constant}$, where $\gamma = C_p/C_v$.
5. **Standing wave (20%):** In an area confined by two walls, similar to the block in the last problem confined between two walls. If you send a right traveling wave from the left wall and a left traveling wave from the right wall, both waves are identical except the traveling direction. You could generate a standing wave. (a) What is the possible wave function of this standing wave? (b) Why it is a standing wave?
6. **Pressure (15%):** (a) Suppose you are driving a submarine in Pacific Ocean, calculate the absolute pressure at an ocean depth of 2.0 km. Let the density of seawater is 1000kg/m^3 and the air above exerts a pressure of 100.0 kPa. (b) In this depth, what pressure/ force will be exerted by the water on a circular window of radius 2.5 m of the submarine? [20+10 = 30]
7. **Kepler's law (20%):** (a) Write down the Kepler's third law of planetary motion. (b) Let the Mars is moving in an elliptical orbit around the Sun. Its distance from the Sun ranges between 0.5 AU to 2.5 AU. Calculate the eccentricity of Mars orbit (c) Find out the period of it around the sun. (Here, 1 AU = one astronomical unit, the average distance from Sun to Earth = 1.496×10^{11} m and $K_s = 2.97 \times 10^{-19} \text{ s}^2/\text{m}^3$).

