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General Physics II, Midterm 2
PHYS10400, Class year 103-2
04-30-2015

Solution

Problems (6 Problems, total 120 points)

1.

The total energy of the projectile-plus-nucleus system is

$$E = \frac{1}{2}mv^2 + \frac{(e)(Ze)}{4\pi\epsilon_0 r}$$

, m is the ~~charge~~ mass of the proton
v is the velocity of the proton
e is the Charge of the proton

If the nucleus remains fixed due to its large mass. then when the proton is at infinite, the total energy is $\frac{1}{2}mv_0^2$

By conservation of energy $\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{eZe}{4\pi\epsilon_0 r}$

At closest point. $v \to 0$, distance is R

$$\frac{1}{2}mv_0^2 = \frac{Ze^2}{4\pi\epsilon_0 R} \rightarrow R = \frac{Ze^2}{4\pi\epsilon_0 (\frac{1}{2}mv_0)^2}$$

2.

2) (a) Orbital angular momentum.

the circular motion's centrifugal force is provided by the electric force between the electron and the nucleus.

$$\frac{m_e v^2}{r} = \frac{Ze^2}{4\pi\epsilon_0 r^2}$$, Z = Charge of the nucleus, e = charge of the electron

Classically, $L = r \times p = r \times mv$, but $r \perp v$.

$\therefore \vec{L} = mvr$

(b) The total classical energy $E = E_K + E_P$.

E_K = the kinetic energy of the electron with velocity.

E_P = the electric potential energy

$$\therefore E = \frac{1}{2} m_e v^2 - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}$$

$$= \frac{1}{2}\frac{Ze^2}{4\pi\epsilon_0 r} - \frac{Ze^2}{4\pi\epsilon_0 r} = -\frac{Ze^2}{4\pi\epsilon_0 (2r)}$$

(c) $$L = m_e v r \rightarrow L^2 = m_e(m_e v^2) r^2 = m_e\left(\frac{Ze^2}{4\pi\epsilon_0 r}\right) r^2$$

$$= \frac{m_e Z e^2 r}{4\pi\epsilon_0}$$

$$\therefore E = -\frac{Ze^2}{4\pi\epsilon_0(2r)} = -\frac{m_e e^4 Z^2}{2(4\pi\epsilon_0)^2 L^2} = -\left(\frac{m_e e^4}{32\pi^2\epsilon_0^2}\right)\frac{Z^2}{L^2}$$

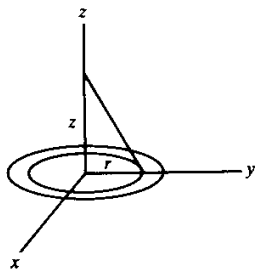
(d) If $L = n\hbar$

$$E = -\left(\frac{m_e e^4}{32\pi^2\epsilon_0^2}\right)\frac{Z^2}{L^2} = -\left(\frac{m_e e^4}{32\pi^2\epsilon_0^2}\right)\frac{Z^2}{\hbar^2 n^2}$$

(e) from (c) $$L^2 = \frac{m_e Z e^2 r}{4\pi\epsilon_0} = n^2\hbar^2 \rightarrow r = \frac{(4\pi\epsilon_0)\hbar^2}{Ze^2 m_e} n^2 = \left(\frac{a_0}{Z}\right) n^2$$

$$a_0 = \frac{(4\pi\epsilon_0)\hbar^2}{e^2 m_e} = 5.29\times10^{-11}\,m = 0.5\,\mathring{A}$$

3.



We consider the charged disk of radius R to lie in the xy plane centered at the origin than calculate the field along the z axis directly, we first calculate the potential, $V(z)$. We consider the disk to be made up of concentric rings, each ring contributing to the potential:

$$dV = \frac{\sigma}{4\pi\epsilon_0}\frac{(2\pi r\,dr)}{(r^2+z^2)^{1/2}}$$

We then integrate from $r = 0$ to $r = R$:

$$V(z) = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{r\,dr}{(r^2+z^2)^{1/2}} = \frac{\sigma}{2\epsilon_0}(r^2+z^2)^{1/2}\Big|_0^R = \frac{\sigma}{2\epsilon_0}[(R^2+z^2)^{1/2} - |z|] \tag{1}$$

Noting that $|z|/(R^2+z^2)^{1/2} = \cos(\alpha/2)$ we have, factoring $|z|$ out of the brackets,

$$V(z) = \frac{\sigma|z|}{2\epsilon_0}\left[\frac{1}{\cos(\alpha/2)} - 1\right]$$

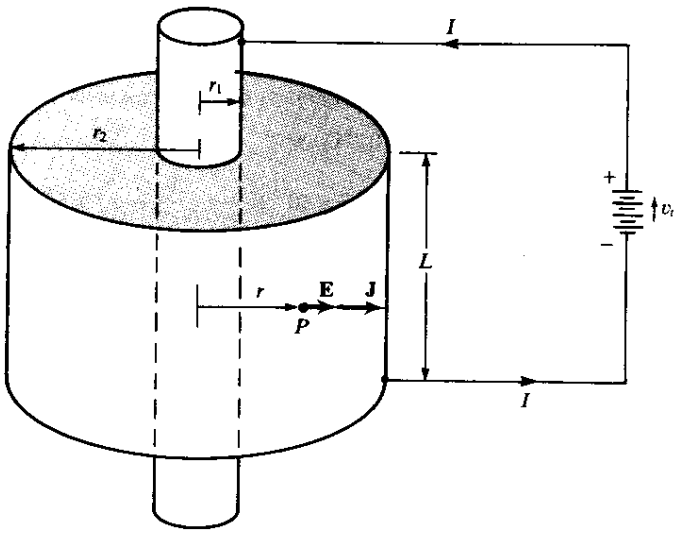
Noting that by symmetry the electric field along the z axis is parallel to that axis, we have, using form (1),

$\mathbf{E}(z) = E_z(z)\mathbf{k}$, with

$$E_z(z) = -\frac{dV(z)}{dz} \equiv -V'(z) = \begin{cases} \dfrac{\sigma}{2\epsilon_0}\left[1 - \dfrac{z}{(R^2+z^2)^{1/2}}\right] & z > 0 \\ -\dfrac{\sigma}{2\epsilon_0}\left[1 + \dfrac{z}{(R^2+z^2)^{1/2}}\right] & z < 0 \end{cases}$$

$$= \begin{cases} \dfrac{\sigma}{2\epsilon_0}\left(1 - \cos\dfrac{\alpha}{2}\right) & z > 0 \\ -\dfrac{\sigma}{2\epsilon_0}\left(1 - \cos\dfrac{\alpha}{2}\right) & z < 0 \end{cases}$$

4.



(a) Assuming radial flow of charge between rod and cylinder, we have at P

$$J = \frac{I}{2\pi r L} \qquad \text{and} \qquad E = \rho J = \frac{\rho I}{2\pi r L}$$

with both **J** and **E** in the direction of **r**. Then, by definition of the potential,

$$dv = -\mathbf{E} \cdot d\mathbf{s} = -E\,dr = -\frac{\rho I}{2\pi L}\frac{dr}{r}$$

and so, noting the polarity of v_t,

$$-v_t = \int_{r_1}^{r_2} dv = -\frac{\rho I}{2\pi L}\int_{r_1}^{r_2}\frac{dr}{r} = -\frac{\rho I}{2\pi L}\ln\frac{r_2}{r_1}$$

Solving for I,

$$I = \frac{2\pi L v_t}{\rho \ln(r_2/r_1)}$$

(b) From ***a***,

$$J = \frac{I}{2\pi r L} = \frac{v_t}{\rho r \ln(r_2/r_1)} \qquad \text{and} \qquad E = \rho J = \frac{v_t}{r \ln(r_2/r_1)}$$

(c) From Ohm's law,

$$R = \frac{v_t}{I} = \frac{\rho \ln(r_2/r_1)}{2\pi L}$$

5. In the figure shown above (Fig II-2), an essential mass spectrometer which can be used to measure the mass of an ion. An ion of mass ***m*** (to be measured) and charge ***q*** is produced in source ***S***. The initially stationary ion is accelerated by the electric field due to a potential difference ***V***. The ion leaves ***S*** and enter a separator chamber in which a uniform magnetic field ***B*** is perpendicular to the path of the ion. The magnetic field causes the ion to move in a semicircle, striking (and thus altering) a detector at a distance ***x*** from the entry slit. Suppose that in a certain trial ***B***=80 ***mT*** and ***V***=1000***V***, and ion charge=+1.6022×10^{-19} ***C*** strike the plate at ***x***=1.6254 ***m***. (a) what is the mass ***m*** of the individual ions? (10%) (b) Why this instrument can only measure the charged particles? (10%)

(a) Using the conservation of mechanical energy $\frac{1}{2}mv^2 = qV$, $\rightarrow$ $v = \sqrt{\frac{2qV}{m}}$

$$F_B = qVB = \frac{mv^2}{r} \rightarrow r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}},$$

But $x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}} \rightarrow m = \frac{B^2qx^2}{8V}$ =3.3863×10^{-25} kg

(b) Only charged particles will subject to magnetic force.

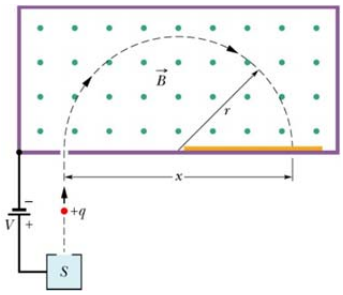
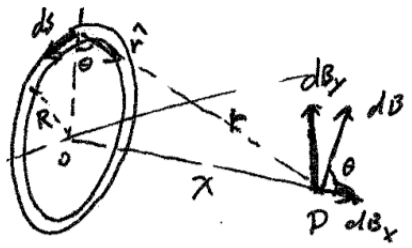


Fig II-2

6.

Circular Current loop



$r^2 = x^2 + R^2$

$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{s} \times \hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{(x^2+R^2)}$$

$$dB_x = dB\cos\theta \rightarrow B_x = \oint dB\cos\theta = \frac{\mu_0 I}{4\pi}\oint \frac{ds\cos\theta}{x^2+R^2}$$

$$\cos\theta = \frac{R}{(x^2+R^2)^{1/2}}$$

$$\therefore B_x = \frac{\mu_0 I}{4\pi}\frac{1}{(x^2+R^2)^{3/2}}\oint ds \qquad \oint ds = 2\pi R$$

$$\therefore B_x = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

$B_y = 0$ symmetric