



Final-2 Solution

1. Solution :

(a) The speed of electric (electromagnetic) field propagation in copper wire is slower than in vacuum by a factor referred to as the velocity factor. The speed of electromagnetic waves propagate in vacuum is **299,792,458 meters per second**. The velocity factor for a 12-gauge copper wire copper wire is about 0.951 (according to this [source](#)). Therefore, the speed of electricity in a 12-gauge copper wire is $299,792,458 \text{ meters per second} \times 0.951$ or 285,102,627 meters per second. This is about **280,000,000 meters per second** which is not very much different from the speed of electromagnetic waves (light) in vacuum. (b) The speed of electric current in copper wire is the average drift velocity of electrons in the wire. This can be calculated using the applet shown below from the [Hyperphysics](#) webpage: [Microscopic View of Electric current](#). For a 12-gauge copper wire carrying a 10-ampere DC current, the average drift velocity is about 80 centimeters per hour or about **0.0002 meters per second**.

2. Solution :

a) The input voltage is $\Delta V_{\text{in}} = IZ = I\sqrt{R^2 + X_C^2} = I\sqrt{R^2 + (1/\omega C)^2}$. The output voltage is

$$\Delta V_{\text{out}} = IX_C = \frac{I}{\omega C}.$$
 The gain ratio is

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{I/\omega C}{I\sqrt{R^2 + (1/\omega C)^2}} = \frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}.$$

(b, c) As $\omega \rightarrow 0$, $\frac{1}{\omega C} \rightarrow \infty$ and R becomes negligible in comparison. Then

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \rightarrow \frac{1/\omega C}{1/\omega C} = \boxed{1}.$$
 As $\omega \rightarrow \infty$, $\frac{1}{\omega C} \rightarrow 0$ and $\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \rightarrow \boxed{0}.$

(d) $\frac{1}{2} = \frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}$ $R^2 + \left(\frac{1}{\omega C}\right)^2 = \frac{4}{\omega^2 C^2}$ $R^2 \omega^2 C^2 = 3$ $\omega = 2\pi f = \frac{\sqrt{3}}{RC}$

$$\boxed{f = \frac{\sqrt{3}}{2\pi RC}}$$

3. Solution:

The electron has to travel in x-direction a total distance l

$$l = Vt \quad t = \frac{l}{v}, \quad t = \text{the time to travel.}$$

$$F_e = qE = ma_y \rightarrow a_y = \frac{qE}{m}$$

The total distance travel in y-direction is

$$\begin{aligned} y &= \frac{1}{2} a_y t^2 = \frac{1}{2} \frac{qE}{m} t^2 \\ &= \frac{1}{2} \frac{qE}{m} \left(\frac{l}{v} \right)^2 = D \end{aligned}$$

$$\therefore \left(\frac{l}{v} \right)^2 = \frac{2Dm}{qE}$$

$$\frac{l}{v} = \sqrt{\frac{2Dm}{qE}}$$

$$V = l \sqrt{\frac{qE}{2Dm}}$$

This means V has to be bigger than this value for electron to "fly through" the electric field region

4. Solution:

In a L-C circuit

$$U \equiv \text{total energy} = U_c + U_L$$

$$= \frac{1}{2C} Q(t)^2 + \frac{1}{2} L I^2$$

$$\frac{dU}{dt} = \frac{d}{dt} \left(\frac{Q}{2C} + \frac{L I^2}{2} \right)$$

$$= \frac{Q}{C} \frac{dQ}{dt} + L I \frac{dI}{dt}$$

$$= \frac{Q}{C} \frac{dQ}{dt} + \frac{dQ}{dt} L - \frac{L d^2Q}{dt^2}$$

$$= 0$$

$$\therefore \frac{Q}{C} + L \frac{d^2Q}{dt^2} = 0$$

$$\text{or } \frac{d^2Q(t)}{dt^2} + \frac{1}{LC} Q = 0$$

$$Q(t) = Q_{\max} \cos(\omega t + \phi), \quad \omega = \frac{1}{\sqrt{LC}}$$

5. Solution:

$$E_1 = \text{light travels to fixed mirror} = E_0 e^{i\omega t}$$

$$E_2 = \text{light travels to the moving mirror} = E_0 e^{i(\omega t + \phi)}$$

$$E = E_1 + E_2 = E_0 [e^{i\omega t} + e^{i(\omega t + \phi)}]$$

$$I = E^* \cdot E = E_0^2 [e^{-i\omega t} + e^{-i(\omega t + \phi)}] [e^{i\omega t} + e^{i(\omega t + \phi)}]$$

$$= E_0^2 [1 + e^{i\phi} + e^{-i\phi} + 1]$$

$$= E_0^2 (2 + e^{i\phi} + e^{-i\phi})$$

$$= E_0^2 (2 + 2 \cos \phi)$$

$$= 2E_0^2 2 \cos^2 \left(\frac{\phi}{2} \right)$$

$$= 4E_0^2 \cos^2 \left(\frac{\phi}{2} \right), \quad \phi = \frac{2\pi d}{\lambda}$$

It is the same as $\because \frac{\lambda}{2\pi} = \frac{d}{x} \Rightarrow x = \text{phase} = \frac{\phi}{2\pi} = \frac{2\pi d}{\lambda}$
 a double-slit Young's interference.