Midterm-1 Exam Question's Solution (2014.10.30)

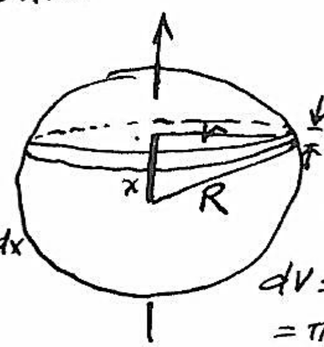
1.

Moment of Inertia of a solid sphere :

Solid sphere about any diameter

Cut the sphere into many horizontal disks of radius r.

$$dI_{disk} = \frac{1}{2} r^2 dm$$



$$dI_{disk} = \frac{1}{2} r^2 dm = \frac{1}{2}(R^2-x^2)\,\pi\rho\,(R^2-x^2)\,dx$$

$$= \frac{\pi\rho}{2}(R^2-x^2)^2\,dx$$

$r = \sqrt{R^2-x^2}$

$dV = \pi r^2 dx$

$= \pi(R^2-x^2)\,dx$

$dm = \rho\, dV = \pi\rho(R^2-x^2)\,dx$

$$I = (2)\frac{\pi\rho}{2}\int_0^R (R^2-x^2)\,dx$$

$$= \frac{8\pi\rho R^5}{15}$$

But $\rho = \frac{M}{V} = \frac{3M}{4\pi R^3}$

$$\therefore I = \left(\frac{8\pi R^5}{15}\right)\left(\frac{3M}{4\pi R^3}\right) = \frac{2}{5}MR^2$$

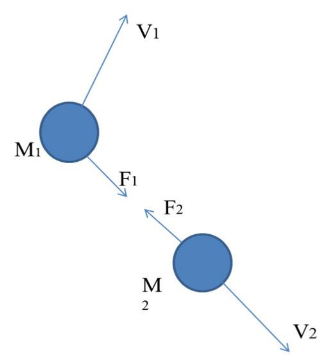
2.

K.E = $\frac{1}{2} mv^2 = \frac{1}{2} M(\dot{\omega}R)^2$ [Since $V=\dot{\omega}R$]

$=1/2\ MR^2\dot{\omega}^2$ [Since $I= MR^2$]

$=1/2\ I\dot{\omega}^2$

3. According to Newton's 3rd law



$F_1 = -F_2$

$F_1 + F_2 = 0$

Or, $M_1 a_1 + M_2 a_2 = 0$

$M_1\, dV_1/dt + M_2\, dV_2/dt = 0$

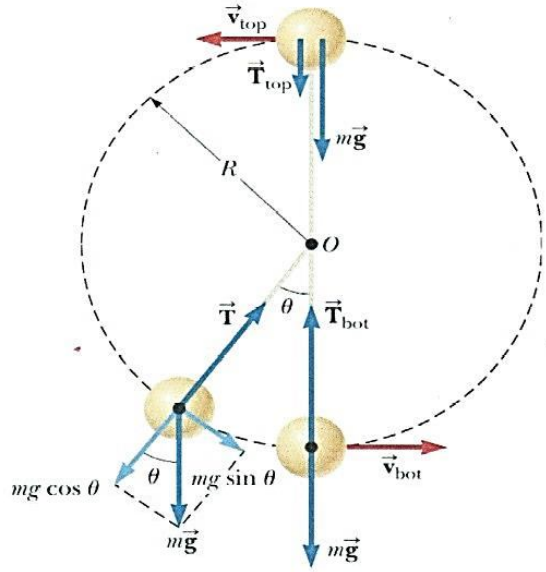
$d(M_1V_1/dt) + d(M_2V_2/dt) = 0$

$d/dt(M_1V_1 + M_2V_2) = 0$

So, $M_1V_1 + M_2V_2 =$ Constant, It means that during the motion this quantity is conserved. Define linear momentum, $P = MV$. Therefore the linear momentum is conserved.

4.

As shown in the figure to the right, (Example 6.6 in the text book) the forces acting on a sphere of mass m connected to a cord of length R and rotating in a vertical circle centered at O. Forces acting on the sphere are shown when the sphere is at the top and bottom of the circle and at an arbitrary location. From the force diagram in Figure on the right, we see that the only forces acting on the sphere are the gravitational force $\mathbf{F_g} = \mathbf{mg}$ exerted by the Earth and the force $\mathbf{T}$ exerted by the cord. We resolve $\mathbf{F_g}$ into a tangential component $\boldsymbol{mg \sin \theta}$ and a radial component $\boldsymbol{mg \cos \theta}$.



Apply Newton's second law to the sphere in the tangential direction:

$$\sum F_g = mg\sin\theta = ma_t, \quad a_t = g\sin\theta$$

Apply Newton's second law to the forces acting on the sphere in the radial direction, noting that both $\mathbf{T}$ and $\mathbf{a_r}$ are directed toward O:

$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$

$$T = mg\left(\frac{v^2}{Rg} + \cos\theta\right)$$

Let us evaluate this result at the top and bottom of the circular path:

$$T_{top} = mg\left(\frac{v_{top}^2}{Rg} - 1\right), \text{ and } T_{bottom} = mg\left(\frac{v_{bottom}^2}{Rg} + 1\right)$$

5.

This problem is from Page 188 (Example 7.9) of text book.

(a) The separation of two atoms is where the potential is in its minimum. To find the minimum, we set $\dfrac{dU(x)}{dx} = 4\varepsilon\dfrac{d}{dx}\left[(\dfrac{\sigma}{x})^{12} - (\dfrac{\sigma}{x})^{6}\right] = 4\varepsilon\left[\dfrac{-12\sigma^{12}}{x^{13}} + \dfrac{6\sigma^6}{x^7}\right] = 0$

$$\frac{dU(x)}{dx} = 4\varepsilon\frac{d}{dx}\left[(\frac{\sigma}{x})^{12} - (\frac{\sigma}{x})^{6}\right] = 4\varepsilon\left[\frac{-12\sigma^{12}}{x^{13}} + \frac{6\sigma^6}{x^7}\right] = 0, \quad x = (2)^{\frac{1}{6}}\sigma$$

(b) Plug in numbers given, x=2.95×10-^{10} m

6.

(a)

$$(1)\ \ m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$(2)\ \ v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

$$(3)\ \ m_1v_{1i} - m_1v_{2i} = -m_1v_{1f} + m_1v_{2f}$$

$$2m_1v_{1i} + (m_2 - m_1)v_{2i} = (m_1 + m_2)v_{2f}$$

$$v_{2f} = \frac{2m_1v_{1i} + (m_2 - m_1)v_{2i}}{m_1 + m_2}$$

$$v_{2f} = \frac{2(1.60\text{ kg})(4.00\text{ m/s}) + (2.10\text{ kg} - 1.60\text{ kg})(-2.50\text{ m/s})}{2.10\text{ kg} + 1.60\text{ kg}}$$

$$= 3.12\text{ m/s}$$

$$v_{1f} = v_{2f} - v_{1i} + v_{2i} = 3.12\text{ m/s} - 4.00\text{ m/s} + (-2.50\text{ m/s})$$

$$= -3.38\text{ m/s}$$

(b)

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$v_{2f} = \frac{m_1v_{1i} + m_2v_{2i} - m_1v_{1f}}{m_2}$$

$$v_{2f}$$

$$= \frac{(1.60\text{ kg})(4.00\text{ m/s}) + (2.10\text{ kg})(-2.50\text{ m/s}) - (1.60\text{ kg})(3.00\text{ m/s})}{2.10\text{ kg}}$$

$$= -1.74\text{ m/s}$$