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General Physics I, Final-1
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Final-1 Solution

1. Solution : *(Similar to 15.4, text book 8th edition)*

(a)For a spring , F= - kx= ma

$$a=\frac{d^2x}{dt^2}=\frac{-kx}{m}$$

$$\frac{d^2x}{dt^2}=-\frac{k}{m}x=\omega^2x \text{ , where } \omega=\sqrt{\frac{k}{m}}$$

$$\frac{d^2x}{dt^2}+\omega^2x=0$$

(b) Here, s = Lθ

So , F= ma = m $\frac{d^2s}{dt^2}=-mg\sin\theta$

$$\frac{d^2s}{dt^2}=-g\sin\theta$$

$$\frac{d^2\theta}{dt^2}=-\frac{g}{L}\sin\theta \qquad \text{Since } s = L\theta$$

(c) $\frac{d^2\theta}{dt^2}=-\frac{g}{L}\theta$, if θ is very small , sinθ=θ

$$\frac{d^2\theta}{dt^2}+\omega^2\theta=0 \quad \text{, where } \omega=\sqrt{\frac{g}{L}}$$

Which is a simple harmonic motion equation

2. Solution: *(Similar to 18.2 , text book 8th edition)*

(a)The wave function of the travelling waves are

$y_1 = A\sin(kx-\omega t)$ (for right)

$y_2 = A\sin(kx+\omega t)$ (for left)

(b) Wave function for the standing wave will be

$$Y= y_1+y_2$$
$$= A\sin(kx-\omega t) + A\sin(kx+\omega t)$$
$$= A[\sin(kx-\omega t)+\sin(kx+\omega t)]$$
$$= 2A\sin kx\cos\omega t \qquad \text{Since } \sin C+\sin D=2\sin\frac{C+D}{2}\cos\frac{D-C}{2}$$

(c)The condition for the node can be found for $2A\sin kx=0$

or , $\sin kx=0$ or, $kx=n\pi$

$$\frac{2\pi x}{\lambda}=n\pi, \qquad \text{or, } x=\frac{n}{2}\lambda \text{ , where } n=0,1,2,3............N$$

So, $x=0,\frac{\lambda}{2},\lambda,\frac{3\lambda}{2},$........................ are the condition for NODE

3. Solution: ***(Similar to page 472 , Halliday Book)***

The internal energy chnage

$\Delta E_{int} = \Delta(nC_vT) = C_v\Delta(nT)$..................................(1)

But we know , $PV = nRT$ for a ideal gas

So , $nT = \dfrac{PV}{R}$

Put this value in equation (1) , we get

$\Delta E_{int} = C_v\Delta(\dfrac{PV}{R})$, But here P,V, R are all constant

So there will no change of internal energy

$\Delta E_{int} = 0$

4. Solution: ***(Similar to 21.5 , text book 8th edition)***

(a) C_p needs to consider extra work done by (or be done to) the gas ,

so extra energy is required.

(b) We know , $\Delta E_{in} = nC_vdT = Q + W = 0 - PdV$ (for adiabatic process)

$nC_vdT = -PdV$ or, $ndT = -\dfrac{PdV}{C_v}$

Now for a ideal gas, PV=nRT

Using differntiation we can write ,

$PdV + VdP = nRdT$

$PdV + VdP = -\dfrac{PdV}{C_v}R = -\dfrac{(C_p - C_v)PdV}{C_v}$, Since $R = C_p - C_v$

$PdV[1 + \dfrac{(C_p - C_v)}{C_v}] = -VdP$

$\dfrac{dV}{V}(1 + \gamma - 1) + \dfrac{dP}{P} = 0$, Since $\dfrac{C_p}{C_v} = \gamma$

Using integration we get, $\ln P + \ln V^{\gamma} = \ln C$, Where C is constant

$\therefore \; PV^{\gamma} = \text{Constant}$

5. Solution: ***(Set-up from the basic)***

(a) We know the gravitational force , $F = \dfrac{GmM_{ins}}{R^2}$

For a spherically symmetric mass, the net gravity force on an object from that mass would be only that due to the mass inside its radius, and that would act as if it were a point mass located at the center.

So we can write, m = object mass , M_{ins} = Mass of Earth inside the radius

$$M_{ins} = \rho V_{ins} = \rho \frac{4\pi R^3}{3} \ , \qquad \because \ V_{ins} = \frac{4\pi R^3}{3}$$

$$So, F = \frac{4\pi Gm\rho}{3} R = kR, \text{ where } k = \frac{4\pi Gm\rho}{3} = \text{constant}$$

Which is similar to $F = -kx$

We can compare its motion with a Simple Harmonic Oscillator (SHO)

So the object motion will be simple harmonic motion and it will reach to the south pole.

(b) Now we know the frequency of oscillator is

$$f = 2\pi\sqrt{\frac{k}{m}} \ ,$$

$$\text{So } \ T = \frac{1}{f} = \frac{1}{2\pi}\sqrt{\frac{m}{k}} = \frac{1}{2\pi}\sqrt{\frac{m}{\frac{4\pi Gm\rho}{3}}} = \sqrt{\frac{3}{4\pi G\rho}}$$