



SN: _____, Name: _____

ABSOLUTELY NO CHEATING!

Please write the answers on the blank space or on the back of this paper to save resources.

1.

(a)

(a)

$$e \equiv \frac{W_{\text{carnot}}}{|Q_{\text{in}}|} = \frac{|Q_{\text{in}}| - |Q_{\text{out}}|}{|Q_{\text{in}}|} = 1 - \frac{|Q_{\text{out}}|}{|Q_{\text{in}}|}$$

In a Carnot cycle, $\Delta E_{\text{in}} = 0$ (path $A \rightarrow B$)

$\therefore |Q_{\text{in}}| = |W_{\text{AB}}|$, $W_{\text{AB}} \equiv$ work done between $A \rightarrow B$

$A \rightarrow B$, $|Q_{\text{in}}| = |W_{\text{AB}}| = nRT_{\text{H}} \ln\left(\frac{V_{\text{B}}}{V_{\text{A}}}\right)$ absorb energy

$C \rightarrow D$, $|Q_{\text{out}}| = |W_{\text{CD}}| = nRT_{\text{C}} \ln\left(\frac{V_{\text{C}}}{V_{\text{D}}}\right)$

$$\therefore \frac{|Q_{\text{out}}|}{|Q_{\text{in}}|} = \frac{T_{\text{C}}}{T_{\text{H}}} = \frac{\ln\left(\frac{V_{\text{C}}}{V_{\text{D}}}\right)}{\ln\left(\frac{V_{\text{B}}}{V_{\text{A}}}\right)} \quad \text{--- ①}$$

$$\text{But } P_i V_i^{\gamma} = P_f V_f^{\gamma} \rightarrow T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\left. \begin{array}{l} B \rightarrow C \quad T_{\text{H}} V_{\text{B}}^{\gamma-1} = T_{\text{C}} V_{\text{C}}^{\gamma-1} \\ D \rightarrow A \quad T_{\text{H}} V_{\text{A}}^{\gamma-1} = T_{\text{C}} V_{\text{D}}^{\gamma-1} \end{array} \right\} \Rightarrow \left(\frac{V_{\text{B}}}{V_{\text{A}}}\right)^{\gamma-1} = \left(\frac{V_{\text{C}}}{V_{\text{D}}}\right)^{\gamma-1}$$

$$\therefore \frac{V_{\text{B}}}{V_{\text{A}}} = \frac{V_{\text{C}}}{V_{\text{D}}} \quad \text{--- ②}$$

$$\text{From ① and ②} \quad \frac{|Q_{\text{out}}|}{|Q_{\text{in}}|} = \frac{T_{\text{C}}}{T_{\text{H}}}$$

$$\Rightarrow e_{\text{carnot}} = 1 - \frac{|Q_{\text{out}}|}{|Q_{\text{in}}|} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \quad *$$

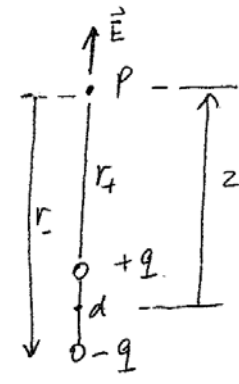
(b)

$$\begin{aligned} (b) \quad \Delta S &= \Delta S_h + \Delta S_c \\ &= \frac{|Q_h|}{T_h} - \frac{|Q_c|}{T_c} \quad \text{But } \frac{|Q_c|}{|Q_h|} = \frac{T_c}{T_h} \text{ as in (a)} \\ &= 0 \quad \text{That's why Carnot engine is a perfect engine} \end{aligned}$$

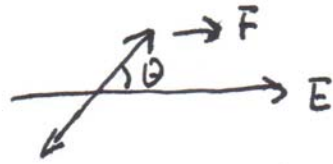
2.

(a)

③ dipole — $\begin{array}{c} +q \qquad -q \\ | \qquad | \\ 0 \text{---} d \text{---} 0 \end{array}$ z
Two charges separated by a distance, have opposite sign are called electric dipole

$$\begin{aligned} \vec{E} &= \vec{E}_{(+)} + \vec{E}_{(-)} = \text{electric field @ pt. P.} \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r_+^2} - \frac{q}{r_-^2} \right] \\ &= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{[z - \frac{1}{2}d]^2} - \frac{1}{[z + \frac{1}{2}d]^2} \right] \\ &= \frac{q}{4\pi\epsilon_0 z^2} \left[\left(1 - \frac{d}{2z}\right)^{-2} - \left(1 + \frac{d}{2z}\right)^{-2} \right], \quad z \gg d, \quad \frac{d}{2z} \ll 1 \\ &= \frac{q}{4\pi\epsilon_0 z^2} \left[\left(1 + \frac{2d}{2z} + \dots\right) - \left(1 - \frac{2d}{2z} + \dots\right) \right] \\ &= \frac{q}{4\pi\epsilon_0 z^2} \frac{2d}{z} \\ &= \frac{1}{2\pi\epsilon_0} \frac{qd}{z^3} \\ &= \frac{1}{2\pi\epsilon_0} \frac{P}{z^3} \quad P \equiv qd = \text{electric dipole moment.} \end{aligned}$$


(b) in page 23-8



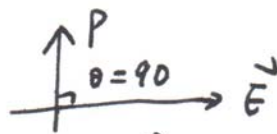
$$\tau = \frac{F}{2} d \sin \theta + F \frac{d}{2} \sin \theta = F d \sin \theta$$

$$= q E d \sin \theta = p E \sin \theta$$

$$\text{in general } \tau = \vec{p} \times \vec{E} = -p E \sin \theta$$

(negative torque)

(c)



$$U = -W = -\int_{90^\circ}^{\theta} \tau d\theta = -\int_{90^\circ}^{\theta} -p E \sin \theta d\theta$$

$$= -p \cdot E \cos \theta = -\vec{p} \cdot \vec{E}$$

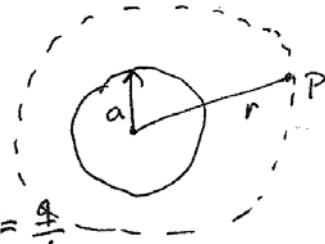
$$\therefore W = \vec{p} \cdot \vec{E}$$

3.

(2) A Spherically Symmetric Charge distribution

(2)-1: A point outside the sphere

Pick a Gaussian surface bigger than the surface



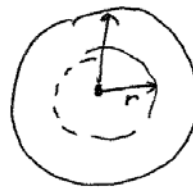
$$\Phi_E = \oint E \cdot dA = E \oint dA = E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = k_e \frac{Q}{r^2} \quad (r > a)$$

(2)-2. A point inside the sphere.

Pick a Gaussian surface as shown

$$q_{in} = \rho V' = \rho \cdot \frac{4}{3}\pi r^3$$

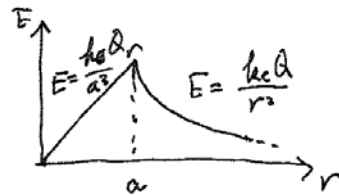


$$\Phi_E = \oint E dA = E \oint dA = E (4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{q_{in}}{4\pi\epsilon_0 r^2} = \frac{\rho (\frac{4}{3}\pi r^3)}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0} r$$

$$\rho = \frac{Q}{\frac{4}{3}\pi a^3}$$

$$\therefore E = \frac{Qr}{4\pi\epsilon_0 a^3} = k_e \frac{Q}{a^3} r \quad (r < a)$$

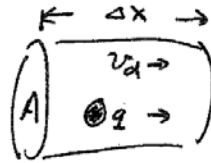


4.

(a) (b)

$n \equiv$ number of mobile charge per unit volume

$$\Delta Q = (n A \Delta x) q$$
$$= (n A v_d \Delta t) q$$



$$I_{av} = \frac{\Delta Q}{\Delta t} = n q v_d A$$

$q \equiv$ charge on each carrier

$v_d \equiv$ carrier speed
 $=$ drift speed

$v_d \equiv$ drift speed.

Example 27.1, page 834

Volume of one mole of Copper $V = \frac{m}{\rho} = \frac{63.5 \text{ g}}{8.95 \text{ g/cm}^3} = 7.09 \text{ cm}^3$

$$n = \frac{6.02 \times 10^{23} \text{ electrons}}{7.09 \text{ cm}^3} \left(\frac{1 \times 10^6 \text{ cm}^3}{1 \text{ m}^3} \right) = 8.49 \times 10^{28} \text{ electrons/m}^3$$

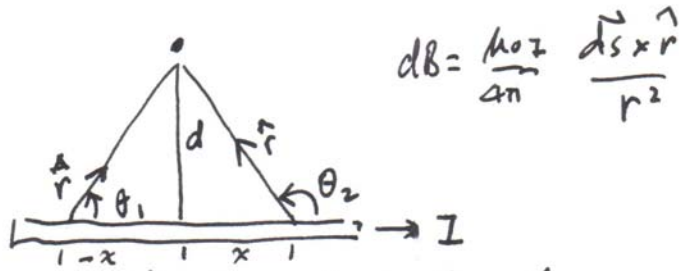
— each copper atom contributes one free electron

$$v_d = \frac{I}{n q A} = \frac{10 \text{ C/s}}{(8.49 \times 10^{28} \text{ e/m}^3) (1.6 \times 10^{-19} \text{ C}) (3.31 \times 10^{-6} \text{ m}^2)}$$
$$= 2.22 \times 10^{-4} \text{ m/s}$$

(c). v_d describe the speed of electrons in the conductor.

When light ~~switched~~ switched on, the electric field that drives the electrons travels through the conductor with a speed close to that of light. So light turns on instantaneously.

5



$$dB = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$\begin{aligned} d\vec{s} \times \hat{r} &= |d\vec{s} \times \hat{r}| \hat{k} \quad \hat{k} \equiv \text{the direction of} \\ &= (ds \sin \theta) \hat{k} \quad \text{the magnetic field} \\ &= (dx \sin \theta) \hat{k} \end{aligned}$$

$\vec{dB} = (dB) \hat{k}$, in this case the magnetic field is pointing out of the paper.

$$= \frac{\mu_0 I}{4\pi} \frac{dx \sin \theta}{r^2}$$

$$r = \frac{a}{\sin \theta} = a \csc \theta$$

$$x = -a \cot \theta$$

$$dx = a \csc^2 \theta$$

$$\therefore dB = \frac{\mu_0 I}{4\pi} \frac{d \csc^2 \theta \sin \theta d\theta}{d^2 \csc^2 \theta}$$

$$(a) \quad B = \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \sin \theta d\theta = \frac{\mu_0 I}{4\pi} (\cos \theta_1 - \cos \theta_2)$$

— for finite size wire
 θ_1 and θ_2 are as indicated
 in the figure

(b) for infinite long wire.

$$\theta_1 = 0, \quad \theta_2 = 180^\circ$$

Plug in the above equation

$$B = \frac{\mu_0 I}{2\pi d} \quad \rightarrow \text{You can also calculate using } \text{Ampere's law} \text{ it will be alot easier}$$