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General Physics I, Final 1
PHYS1000AA, Class year102
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SN:_______________, Name:________________

Final 1 Solution

Chapter 10-22, Serway; ABSOLUTELY NO CHEATING!
Please write the answers on the blank space or on the back of this paper to save resources.

1.

1. (a) $F = -kx = m a_x$

$$a_x = -\frac{k}{m} x$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = -\frac{k}{m} x$$

$$\therefore \frac{d^2x(t)}{dt^2} = -\frac{k}{m} x(t).$$

(b) $F_T = -mg\sin\theta = m a_s = m\frac{d^2s}{dt^2}$

其中 $s = L\theta$ $\quad \frac{d^2s}{dt^2} = L\frac{d^2\theta}{dt^2}$

$$\Rightarrow -mg\sin\theta = mL\frac{d^2\theta}{dt^2}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = \left(-\frac{g}{L}\right)\sin\theta$$

(c) in this figure $s = R\theta$ $\quad \theta = \frac{s}{R}$. $\ddot{\theta} = \frac{\ddot{s}}{R}$

$$\therefore \ddot{\theta} + \left(\frac{g}{L}\right)\theta = 0$$

$$\Rightarrow \frac{\ddot{s}}{R} + \left(\frac{g}{L}\right)\frac{s}{R} = 0$$

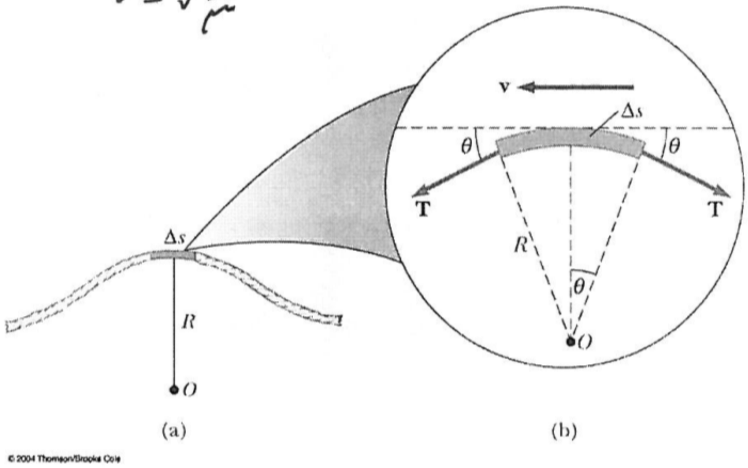
$$\ddot{s} + \left(\frac{g}{L}\right)s = 0$$

this is the same as in (a). So they both are Simple harmonic Oscillators

2.

The speed of waves on strings

$$v = \sqrt{\frac{T}{\mu}}$$



$$F_r = 2T\sin\theta \simeq 2T\theta$$

$$m = \mu\Delta S = \mu R 2\theta = 2\mu R\theta$$

But $F_r = ma = \frac{mv^2}{R} = 2T\theta$

$$\therefore 2T\theta = \frac{mv^2}{R} = \frac{2\mu R\theta v^2}{R}$$

$$\rightarrow v = \sqrt{\frac{T}{\mu}}$$

3.

Adiabatic process for an ideal gas

$Q = 0$.

$$\Delta E_{int} = \cancel{Q} + W = \overset{W}{\cancel{Q}} = n C_v dT = -p dV$$

But $pV = nRT$

$$p dV + V dp = nR dT$$

$$= -\frac{R}{C_v} p dV$$

$$\frac{dV}{V} + \frac{dp}{p} = -\left(\frac{C_p - C_v}{C_v}\right)\frac{dV}{V}$$

$$= (1-\gamma)\frac{dV}{V}$$

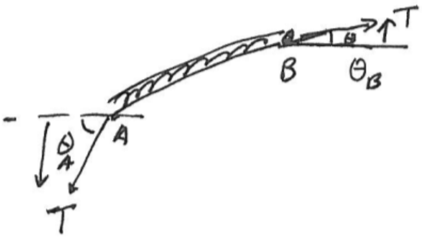
$$\frac{dp}{p} + \gamma\frac{dV}{V} = 0$$

$$\ln p + \gamma \ln V = \text{Constant}$$

$$pV^{\gamma} = \text{constant}$$

4.

Net force in $\hat{y}$ direction



$$\sum F_y = T\sin\theta_B - T\sin\theta_A$$

$$= T(\sin\theta_B - \sin\theta_A) \quad \text{if } \theta \text{ is small}, \sin\theta \approx \tan\theta$$

$$\approx T(\tan\theta_B - \tan\theta_A)$$

$$= T\left(\frac{\partial y}{\partial x}\Big|_{\text{at } B} - \frac{\partial y}{\partial x}\Big|_{A}\right) = m a_y = \mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right)$$

$$\therefore \mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right) = T\left[\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A\right]$$

$$\text{or } \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{(\partial y/\partial x)_B - (\frac{\partial y}{\partial x})_A}{\Delta x}$$

$$= \frac{\partial^2 y}{\partial x^2} \qquad \text{Note: } \frac{\partial f}{\partial x} = \lim_{\Delta x \to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\therefore \boxed{\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}} \quad \text{or} \quad \boxed{\frac{\mu}{T}\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}}$$

$$\Rightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 A\sin(kx - \omega t)$$

$$\frac{-\omega^2\mu}{T}\sin(kx - \omega t) = -k^2\sin(kx - \omega t)$$

$$\rightarrow k^2 = \frac{\mu}{T}\omega^2, \quad v = \frac{\omega}{k}$$

$$\therefore v^2 = \frac{\omega^2}{k^2} = \frac{T}{\mu} \quad \rightarrow \boxed{v = \sqrt{\frac{T}{\mu}}}$$

5.

- in x direction $E_{int} = \frac{1}{2} m V_x^2 + \frac{1}{2} k x^2$ (2 degree of freedom)

$\therefore$ in x, y, z total 6 degree of freedoms

$$E_{int} = N \times 6 \times \frac{1}{2} k_B T = 3 N k_B T = 3 n R T$$

$$C_V = \frac{1}{n} \frac{d}{dT} (E_{int}) = \frac{1}{n} \frac{d}{dT} (3 n R T) = 3 R$$

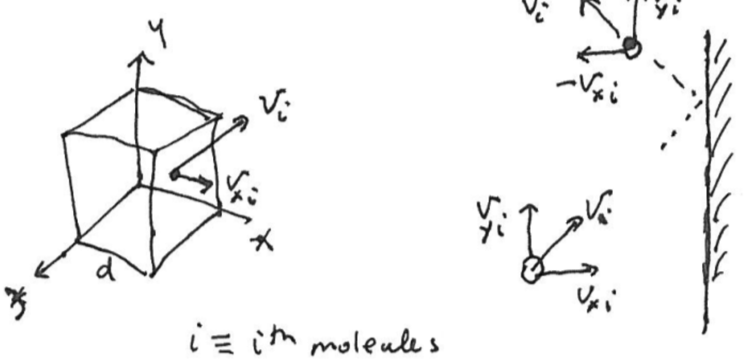
Note : this is OK in high temperature, not low temperature

6.

Please see the class Note - chapter 17

7.

Pressure of N moleules in volume V.



$i \equiv i^{th}$ moleules

x- momentum change

$$\Delta P_{xi} = - m V_{xi} - m V_{xi} = -2 m V_{xi}$$

$$F_i \, \Delta t = \Delta P_{xi} = -2 m V_{xi}$$

$$\Delta t = \frac{2d}{V_{xi}}$$ = time interval between two collisions with the same wall.

$\therefore\ \bar{F}_i \Delta t = -2mV_{xi}$, $\bar{F} \equiv$ Average force component for molecule to move across the cube and back

$\Rightarrow$ long time average force on the molecule

$$\bar{F}_i = \frac{-2mV_{xi}}{\Delta t} = \frac{-2mV_{xi}^2}{2d} = \frac{-mV_{xi}^2}{d}$$

$$\bar{F}_{i,\text{on wall}} = -\bar{F}_i = \frac{mV_{xi}^2}{d}$$

Total average force $\bar{F} = \sum_{i=1}^{N} \frac{mV_{xi}^2}{d} = \frac{m}{d}\sum_{i=1}^{N} V_{xi}^2$

$$\overline{V_x^2} = \frac{\sum_{i=1}^{N} \cancel{V}_{xi}^2}{N} = \text{Average } V_x^2$$

$$\therefore\ F = \frac{m}{d} N \overline{V_x^2}$$

$$V_i^2 = V_{xi}^2 + V_{yi}^2 + V_{zi}^2 \rightarrow \overline{V^2} = \overline{V_x^2} + \overline{V_y^2} + \overline{V_z^2}$$

$$\therefore \rightarrow \overline{V^2} = 3\overline{V_x^2}$$

Thus $F = \frac{N}{3}\left(\frac{m\overline{v^2}}{d}\right)$

$$P = \frac{F}{A} = \frac{F}{d^2} = \frac{1}{3}\frac{N}{d^3} m\overline{v^2} = \frac{1}{3}\left(\frac{N}{V}\right) m\overline{v^2}$$

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\overline{v^2}\right) \sim \left(\frac{N}{V}\right)\left(\frac{1}{2}m\overline{v^2}\right)$$