

SN: _____, Name: _____

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are allowed.

The followings are some useful mathematics you may use without proof in answering your problems.

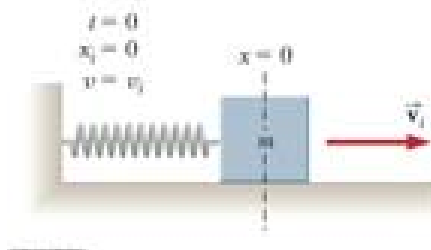
$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots \quad \text{Time average} \quad \overline{x(t)^n} = \langle x(t)^n \rangle = \frac{1}{T} \int_0^T x(t)^n dt$$

For a second order differential equation, $\frac{d^2x}{dt^2} + ax = 0$, the general solution of this equation is

$x(t) = x_0 \cos(at + \phi)$, where x_0 is the maximum, and ϕ is the phase angle.

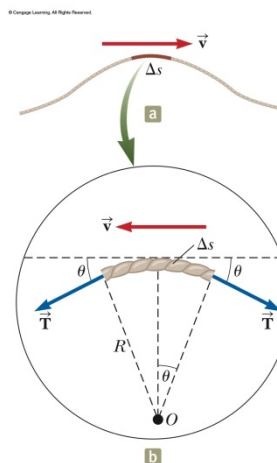
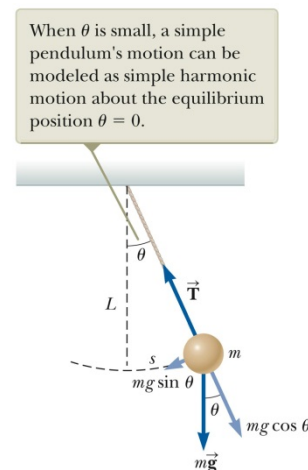
$N_A = 6 \times 10^{23}$, $R = \text{Gas constant} = 8.31 \text{ J/mole K}$, room temperature $= 300\text{K}$, $1\text{atm} = 1.01 \times 10^5 \text{ Pa}$.

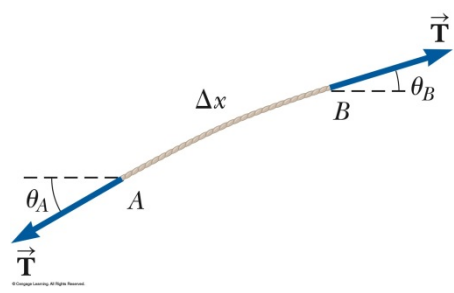
$$\overline{v_x} = \frac{v_{rms}}{\sqrt{3}} \text{ for ideal gas.}$$



Problems (7 Problems, total 120%)

- 1. Harmonic Oscillation (20%):** (a, b) Refer to the figure to the right, write down differential equations that can describe the motion for a mass m attached to a spring of force constant k and the same mass attached to a pendulum of length L . In both cases, use the given parameters. In the upper case, describe the motion in terms of its displacement x ; while in the lower case, describe its motion in terms of the angle θ . (c) If we treat the pendulum as a simple harmonic oscillator and look at only the horizontal displace s at small angle, what will the equation look like?
- 2. Speed of waves on String (15%):** Consider a string as shown in the figure to the right, a string wave is set up in the string and the wave travels to the right. What is the speed of the wave, in terms of the parameters to the right?
- 3. Adiabatic Process: (15%)** Prove that for an adiabatic expansion of an ideal gas, $PV^\gamma = \text{constant}$, where $\gamma = C_p/C_v$.



4. **Wave Equation:** (15%) Continue the last problem, but use the figure to the right to derive the wave equation for a string. If you focused on this section of the string, you can find the mass of the string is oscillating vertically (y-direction) that is it is perpendicular to the wave's travelling direction (say, to the right or in the +x direction). Let the same section, suppose the vibration of the string can be represented as a function $y(x, t)$; a function of both x and t . Prove (or derive) that the wave equation describing this wave motion is $\frac{\mu}{T} \frac{\partial^2 y(x, t)}{\partial t^2} = \frac{\partial^2 y(x, t)}{\partial x^2}$, where μ is the mass density and T is the tension in the string.
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5. **Dulong-Petit law:** (15%) Dulong-Petit law applies to general solid phase of material at high temperature. The molecular specific law of all solids reaches $3R$ (R is the gas constant) at high temperature. The law can be explained using the equipartition energy theorem; that is in each degree of freedom, the average energy shares $1/2 k_B T$. Prove that $C_v = 3R$ in solid in high temperature.
6. **Doppler Effect (20%):** A sound source emits a stable sound of frequency f and sound speed V . A detector is able to detect the sound that can be moving or can be stationary. (a) If the detector is moving toward to source with a speed V_D , and (b) if the detector is moving at the same speed but away from the source; in each case, what are the detected frequencies?
7. **Molecular model of an ideal gas (20%, extra credit):** In an ideal gas model, we treat all molecules identical; molecules interact with one another via short range forces, and make elastic collision with the container walls. Suppose we have N molecules, each has a mass m , velocity V , are confined in a cubic container with each side length d , what is the pressure of this container?