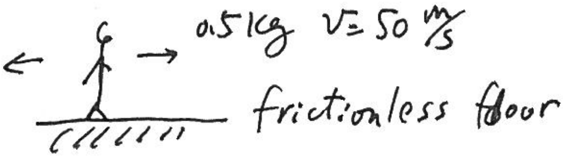
Chap 9 Linear Momentum + Collisions.

1. The energy concepts (Kinetic and potential) help us deal with mechanical motions. However, direction is not considered.



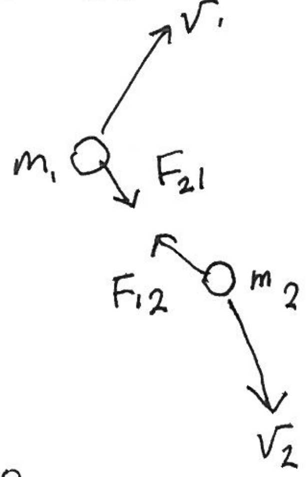
The archer slides backward, this can not be solved using energy methods

→ define linear Momentum

According to Newton's 3rd law

$$F_{12} = -F_{21}$$

$$F_{12} + F_{21} = 0$$



$$\rightarrow m_2 a_2 + m_1 a_1 = 0$$

$$m_1 \frac{dv_1}{dt} + m_2 \frac{dv_2}{dt} = 0$$

$$\frac{d(m_1 V_1)}{dt} + \frac{d(m_2 V_2)}{dt} = 0$$

$$\frac{d}{dt}\left(m_1 V_1 + m_2 V_2\right) = 0 \qquad (9.1)$$

$$m_1 V_1 + m_2 V_2 = \text{Constant}$$

— during the motion this quantity is conserved.

∴ define. $P \equiv mv$ linear Momentum

Now use Newton's 2^{nd} law

$$\sum F = ma = m \frac{dv}{dt}$$

$$= \frac{d}{dt}(mv)$$

$$\sum F = \frac{d}{dt} P$$ → The time rate change of a linear momentum of a particle is equal to the net force acting on the particle.

$(9.1) \rightarrow$ $$\frac{d}{dt}(m_1 v_1 + m_2 v_2) = 0$$

$$\frac{d}{dt}(P_1 + P_2) = 0$$

$$\therefore P_1 + P_2 = \text{Constant} = P_{total}$$

or $$\underline{P_{1i} + P_{2i} = P_{2f} + P_{1f}}$$ → the total momentum of an isolated at all times is equal to its initial momentum

<u>9.2</u> Impulse and Momentum

$$F = \frac{d}{dt} P = \frac{dP}{dt} \rightarrow dP = F\,dt$$

or $$\Delta P = P_f - P_i = F\,dt$$

$$\int_{P_i}^{P_f} dP = \int F\,dt$$

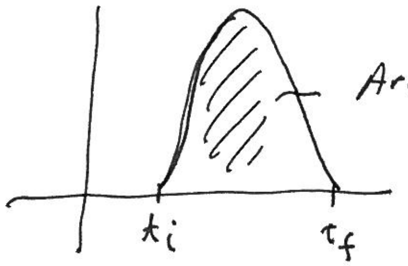
$$\therefore P_f - P_i = \int_{t_i}^{t_f} F\,dt$$ impulse of a force acting on a particle over the time period t_i to t_f

$$I = \int_{t_i}^{t_f} F\,dt$$

Time average force

$$\bar{F} \equiv \frac{1}{\Delta t}\int_{t_i}^{t_f} F\,dt$$

$$I = \bar{F}\Delta t$$



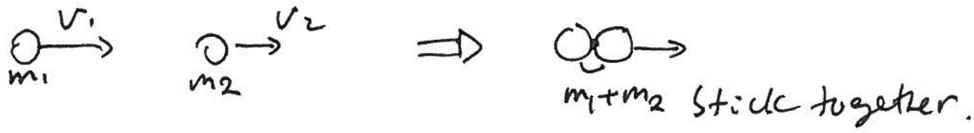
Area $= \int_{t_i}^{t_f} F\,dt$

9.3 Collision in one dimension

Collision : $\begin{cases}\text{direct contact} \\ \text{indirect, not necessary contact.}\end{cases}$

Elastic collision : the total kinetic energy (and total momentum) of the system is conserved.

Inelastic collision : total energy is not conserved
(the momentum maybe conserved)



1) Perfect inelastic collision

$$m_1 v_{1i} + m_2 v_{2i} = (m_1+m_2)v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1+m_2}$$

2) Elastic collision

moment conservation : $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ (9.15)

K. energy conservation : $\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$ (9.16)

(9.16)

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$$

$$m_1(v_{1i} + v_{1f})(v_{1i} - v_{1f}) = m_2(v_{2f} + v_{2i})(v_{2f} - v_{2i})$$

(9.15) $\rightarrow$ $$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$$

$$\therefore \quad v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$\boxed{v_{1i} - v_{2i} = -(v_{1f} - v_{2f})} \quad \text{— (9.19)}$$

Check (9.15) $\cancel{m_1(v_{1i} -} v$

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}} \quad (9.15)$$

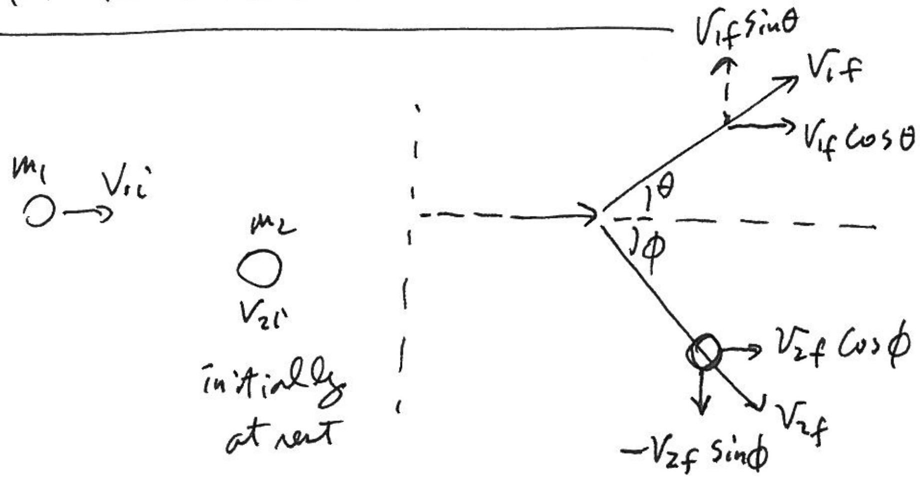
only v_{1f} and v_{2f} are unknown

We can solve these two simultaneously equations and obtain

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

9.4 Two dimentional Collisions



$$m_1 V_{1ix} + m_2 V_{2ix} = m_1 V_{1fx} + m_2 V_{2fx}$$
$$m_1 V_{1iy} + m_2 V_{2iy} = m_1 V_{1fy} + m_2 V_{2fy}$$

Momentum Conservation

if m_2 is initially at rest

$$m_1 V_{1i} = m_1 V_{1f}\cos\theta + m_2 V_{2f}\cos\phi$$ — x direction
$$0 = m_1 V_{1f}\sin\theta - m_2 V_{2f}\sin\phi$$ — y-direction

$$\frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

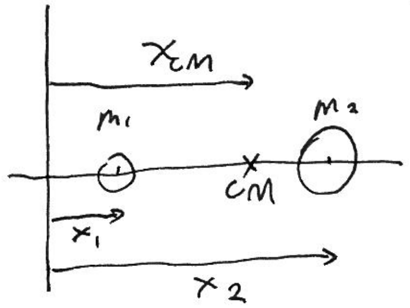
Kinetic energy Conservation

9.5. Center of Mass

$$\sum F_{ext} = M\vec{a}$$

if the whole system's mass were concentrated on a point.

→ As if the external net force were act on a single point at the Center of Mass



$$(m_1 + m_2) X_{CM} = m_1 X_1 + m_2 X_2$$

$$X_{CM} \equiv \frac{m_1 X_1 + m_2 X_2}{m_1 + m_2}$$

$$\therefore x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots m_n x_n}{m_1 + m_2 + \cdots + m_n}$$

$$= \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\sum_i m_i x_i}{M} \quad (9.28)$$

$M \equiv$ total mass

Similarly

$$y_{cm} = \frac{\sum_i m_i y_i}{M}, \quad z_{cm} = \frac{\sum_i m_i z_i}{M}$$

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$= \frac{\sum_i m_i x_i \hat{i} + \sum_i m_i y_i \hat{j} + \sum_i m_i z_i \hat{k}}{M}$$

$$= \frac{\sum_i m_i r_i}{M}, \quad r_i \equiv x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$$

Note. (9.28)

$$x_{cm} = \frac{\sum_i m_i x_i}{M} \approx \frac{\sum_i x_i \Delta m_i}{M}$$

$$= \lim_{\Delta m_i \to 0} \frac{\sum_i x_i \Delta m_i}{M}$$

$$= \frac{1}{M}\int x\,dm \quad - (9.31)$$

Similarly. $y_{cm} = \frac{1}{M}\int y\,dm$,

$$z_{cm} = \frac{1}{M}\int z\,dm$$

or $\vec{r}_{cm} = \frac{1}{M}\int r\,dm$

$$\therefore \int_{v_i}^{v_f} dv = -v_e \int_{M_i}^{M_f} \frac{dM}{M}$$

$$\boxed{v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)}$$ rocket

$$\text{Thrust} = M\frac{dv}{dT} = \left| v_e \frac{dM}{dt} \right|$$

9.7 Rocket propulsion

The propulsion of a rocket is different than that of a car or locomotives, as there is no friction to provide driving force.

— Momentum conservation

The rocket moves in space ejecting gas. The gases are given momentum when they are ejected out of the engine.
The rocket receives a compensating momentum in the opposite direction.

Total magnitude of momentum of rocket + fuel

$(M+\Delta m)v$, v is relative to earth

The rocket ejects a mass Δm, over a short time Δt
rocket speed after the ejection is $v+\Delta v$
the fuel is ejected with a speed v_e relative to the rocket
the fuel's velocity relative to a stationary frame is $v-v_e$

$V_{es} = V_{er} + V_{rs}$
$= -V_e + V$
$= V - V_e$

① $\overleftarrow{v_e}\ \text{o}\ \overrightarrow{V_{rocket}}$

$\therefore \quad V_{es} = V - V_e$

V_e = fuel relative to the rocket

② total momentum conservation

$$(M+\Delta m)v = M(v+\Delta v) + \Delta m(v - v_e)$$

$\rightarrow \quad M\Delta v = v_e \Delta m$

if Δv and Δm are both small.

$M dv = V_e dm \qquad$ But $dm = -dM$

$= v_e(-dM)$

$\therefore \quad dv = \frac{1}{M} v_e(-dm)$

P9.71 The force exerted by the table is equal to the change in momentum of each of the links in the chain.

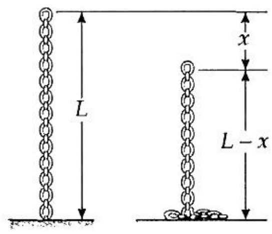


FIG. P9.71

By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}.$$

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0.$$

Since the mass per unit length is uniform, we can express each link of length dx as having a mass dm:

$$dm = \frac{M}{L}dx.$$

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements dm.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance x, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}.$$

The links already on the table have a total length x, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}.$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}.$$

That is, *the total force is three times the weight of the chain on the table at that instant.*