Chap 25 Electric potential

25.1 potential difference and electric potential

for a small displacement ds
work done by the electric force

$$F \cdot ds = q_0 E \cdot ds$$

$dU = -dW$

$\rightarrow dU = -q_0 E \cdot ds$

$\Delta U = U_B - U_A$ (Charge moves from Point A to point B)

$= -q_0 \int_A^B E \cdot ds$

— Because the force $q_0 E$ is conservative the line integral does not depend on the path of from A to B

define potential energy $U = 0$ at one point

also $V \equiv \dfrac{U}{q_0}$ = electric potential (potential

Potential difference $\Delta V = V_B - V_A$

$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \vec{E} \cdot d\vec{s}$$

— Scalar product
— independent of the charge placed in the field.

When an external agent moves a test charge from A to B

W = done by this external agent

$= q\Delta V$

$$1V \equiv 1\frac{J}{C}$$

25.2 potential Difference in a Uniform Electric field.

— How to create a uniform E field

$$V_B - V_A = \Delta V = -\int_A^B \vec{E}\cdot d\vec{s} = -\int_A^B E\cos 0^\circ ds$$

$$= -\int_A^B E ds$$

$$= -Ed \qquad \text{if } \int_A^B ds = d \text{ . E is Uniform}$$

Potential energy Change

$$\Delta U = q_0 \Delta V = -q_0 E d$$

equipotential surface ≡ any surface consisting of a continuous distribution of points having the same electric potential

25.3 Electric potential and potential energy due to point charges

$$V_B - V_A = -\int_A^B E\cdot ds \quad , \vec{E} = k_e \frac{q}{r^2}\hat{r}$$

$$\vec{E}\cdot d\vec{s} = k_e \frac{q}{r^2}\hat{r}\cdot d\vec{s}$$

$$= k_e \frac{q}{r^2} ds\cos\theta \quad . \quad \theta = \text{the angle between } d\vec{s} \text{ and unit vector } \hat{r}$$

$$\therefore V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \left.\frac{k_e q}{r}\right]_{r_A}^{r_B}$$

$$= k_e q \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

— Usually we choose $V = 0$ @ $r \to \infty$

$$\boxed{V \equiv k_e \frac{q}{r}}$$

$$V = k_e \sum_i \frac{q_i}{r_i} \quad \text{for several point charges}$$

Check Fig 25.8. 25.9 page 769

Potential energy due to two charges q_1 and q_2 at a distance r_{12}

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

three charges $$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

25.4

$$dV = -E \cdot ds$$

$$\therefore E = -\frac{dV}{dx} \qquad \text{Similar to} \quad F = -\frac{dU}{dx}$$

$$U = q_0 V \text{ electric}$$

When a test charge is moving ds along an equipotential surface, then $dV = 0$, since V is constant

$$dV = -\vec{E} \cdot d\vec{s} = 0 \quad d\vec{s} \perp \vec{E}$$

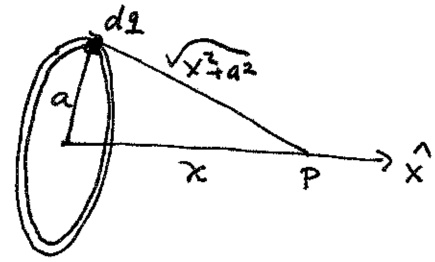
$\therefore$ the equipotential surface must perpendicular to the electric field line.

$$E_r = -\frac{dV}{dr}$$

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

25.5 Electric potential due to continuous charge distribution.

1) Charged ring



$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2+a^2}}$$

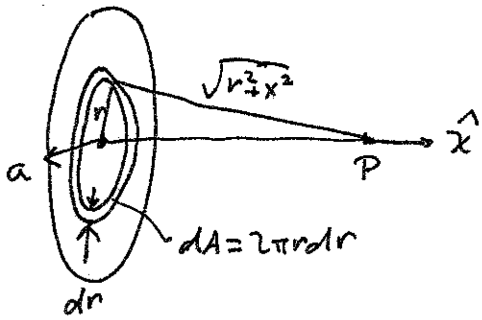
$$= k_e \frac{1}{\sqrt{x^2+a^2}} \int dq$$

$$= k_e \frac{Q}{\sqrt{x^2+a^2}}$$

$$E_x = -\frac{dV}{dx} = -k_e Q \frac{d}{dx}\left(\frac{1}{\sqrt{x^2+a^2}}\right)$$

$$= k_e \frac{Qx}{(x^2+a^2)^{3/2}}$$

② Uniform charged disk



$$dV = \frac{k_e\, dq}{\sqrt{r^2+x^2}} = \frac{k_e \sigma 2\pi r dr}{\sqrt{r^2+x^2}}$$

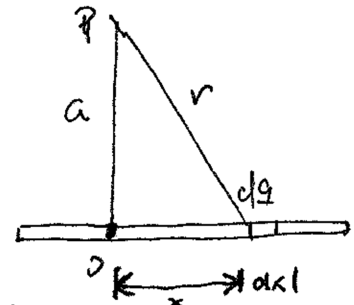
$$\therefore V = \pi k_e \sigma \int_0^a \frac{2r dr}{\sqrt{r^2+x^2}}$$

$$= 2\pi k_e \sigma \left[(x^2+a^2)^{\frac{1}{2}} - x \right]$$

$$E_x = -\frac{dV}{dx} = 2\pi k_e \sigma \left(1 - \frac{x}{\sqrt{x^2+a^2}}\right)$$

③ finite line of charge

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda dx}{\sqrt{x^2+a^2}}$$



$$V = k_e \lambda \int_0^l \frac{dx}{\sqrt{x^2+a^2}}$$

$$= k_e \frac{Q}{l} \int_0^l \frac{dx}{\sqrt{x^2+a^2}}$$ (see Appendix B)

$$= k_e \frac{Q}{l} \ln\left(\frac{l+\sqrt{l^2+a^2}}{a}\right)$$

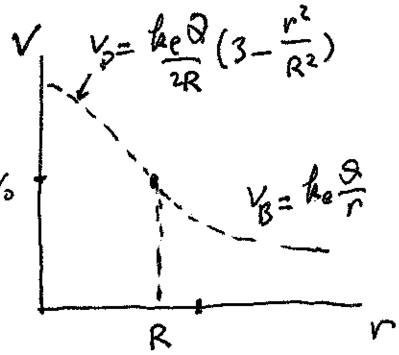
Note: $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right)$

④ Uniform Charged Sphere. Check page 777

25.6 Electric potential due to a Charged Conductor

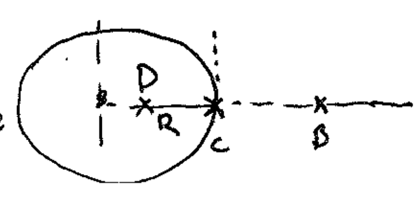
Ex: 25.8

P. 777



$V_0 = \frac{3 k_e Q}{2R}$

Uniformly charged sphere insulator



$$E_r = \frac{kQ}{R^3} r \quad (r < R)$$

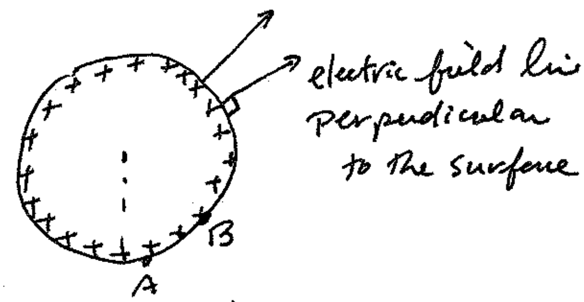
$$V_D - V_C = -\int_R^r E_r\, dr = -\frac{kQ}{R^3}\int_R^r r dr$$

$$= \frac{kQ}{2R^3}(R^2 - r^2)$$

$$V_C = \frac{kQ}{R}$$

$$\therefore V_D = \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

Charged sphere conductor has all their Change on the surface

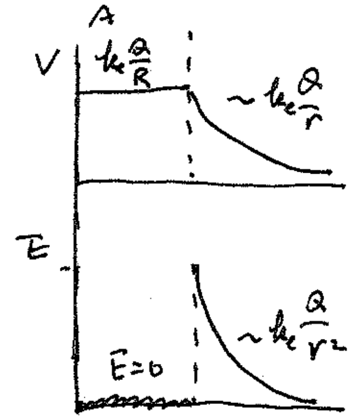


$$V_B - V_A = -\int_A^B E \cdot ds = 0$$

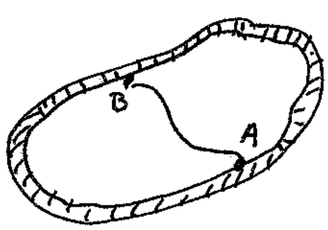
— The Surface is at equipotential

$$V_{\text{on surface}} = k_e \frac{Q}{R}$$

But inside the sphere $\cdot\ \bar{E} = 0$



A cavity within a Conductor



$$V_B - V_A = -\int_A^B E \cdot ds = 0$$

$\bar{E} = 0$. $\bar{E}$ is zero everywhere inside the cavity.