

SN: _____, Name: _____

Solutions:

1. (a) The charge will evenly distributed on the surface of the sphere, since the sphere is a conductor, charges will repel one another. As a result, the charges will spread evenly on surface. (b) Follow (a) the charge density will be $\frac{Q}{4\pi R^2}$

2.

(a) Electric field for a charged disk

$$dq = \sigma dA = \sigma 2\pi r dr$$

$$dE = \frac{z \sigma 2\pi r dr}{4\pi \epsilon_0 (z^2 + r^2)^{3/2}}$$

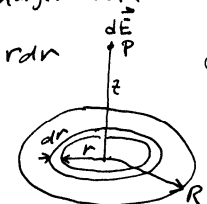
$$= \frac{\sigma z}{4\epsilon_0} \frac{2r dr}{(z^2 + r^2)^{3/2}}$$

$$E = \int_{r=0}^{r=R} dE = \frac{\sigma z}{4\epsilon_0} \int_0^R \frac{2r dr}{(z^2 + r^2)^{3/2}}$$

$$= \frac{\sigma z}{4\epsilon_0} \left[\frac{(z^2 + r^2)^{-1/2}}{-1/2} \right]_0^R = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + R^2}} \right]$$

(b) ① When $R \rightarrow \infty$, $E \rightarrow \frac{\sigma}{2\epsilon_0}$ just like a big plane with uniform charge density

② When $z \rightarrow \infty$, $E \rightarrow 0$, similar to the case of point charge at infinite far away



3.

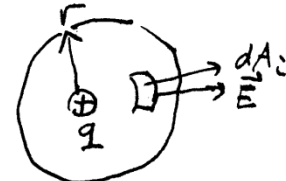
- (a) Gauss law - the relation between the net electric flux through a closed surface and the charge enclosed by the surface
i.e. $\Phi_E = \frac{q}{\epsilon_0}$. $\Phi_E \equiv$ the electric flux

- (b) Take a point charge for example

$$\vec{E} \cdot d\vec{A}_i = E dA_i$$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E dA = E \oint dA$$

$$\vec{E} = k_e \frac{q}{r^2}$$

$$\therefore \Phi_E = k_e \frac{q}{r^2} \cdot 4\pi r^2 = 4\pi \frac{1}{4\pi \epsilon_0} q \therefore \Phi_E = \frac{q}{\epsilon_0}$$


4.

4

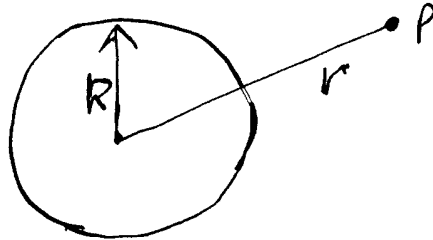
(a) Outside the sphere

$$\Phi_E = \oint E \cdot dA = E \oint dA$$

$$= E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\therefore E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad (\text{for } r > R), \text{ just like a point charge}$$

$$E \propto \frac{1}{r^2}$$

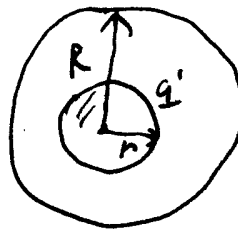


(b) Inside the sphere, $r < R$
Since this is a non-conductor

$$q'_{in} = \rho v' = \rho \cdot \frac{4}{3} \pi r^3$$

$$\vec{E} \cdot \vec{A} = \Phi_E = E \cdot \frac{q'_{in}}{4\pi\epsilon_0 r^2} = \frac{\rho (\frac{4}{3} \pi r^3)}{4\pi\epsilon_0 r^2}$$

$$\therefore \vec{E} = \frac{\rho}{3\epsilon_0} \vec{r} \quad \therefore E \propto r$$



5.

- (a) Entropy : definition $ds = \frac{dQ_r}{T}$
 : Entropy is a measure of disorder
 : (There are many ways to describe)
 : etc.

- (b) In a Carnot cycle. The total change in entropy in one full cycle is $\Delta S = \Delta S_h + \Delta S_c$

$$= \frac{|Q_h|}{T_h} - \frac{|Q_c|}{T_c}$$

But $\frac{|Q_c|}{|Q_h|} = \frac{T_c}{T_h}$

$\therefore \Delta S = 0$ in a Carnot cycle

- (c) Since $\Delta S \geq 0$ in all thermal dynamic systems.
 $\Delta S = 0$ means a Carnot cycle is the most efficient engine, that is, it will transfer all energy to work

(d) $m_c c \Delta T_c = -m_h c \Delta T_h$
 $m_c (T_f - T_c) = -m_h (T_f - T_h)$ $T_f \equiv$ final temperature

$$\rightarrow T_f = \frac{m_c T_c + m_h T_h}{m_c + m_h}$$

(e)
$$\Delta S = \int_c \frac{dQ_c}{T} + \int_h \frac{dQ_h}{T} = m_c \int_{T_c}^{T_f} \frac{dT}{T} + m_h \int_{T_h}^{T_f} \frac{dT}{T}$$

$$= m_c \ln\left(\frac{T_f}{T_c}\right) + m_h \ln\left(\frac{T_f}{T_h}\right)$$