Chap 16 Wave Motion

16.1 propagation of a disturbance

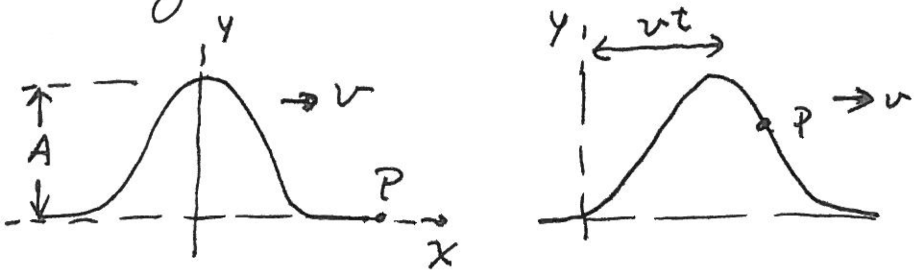
Mechanical wave : (1) Source of disturbance
(2) Medium
(3) Mechanism through which the medium's elements can influence each other

transverse wave : the medium move perpendicular to the traveling of the wave

logitudinal wave : " parallel " .

Earthquake: Longitudinal (faster, 7 to 8 km/s near the surface)
P-wave, primary wave
transverse (slower, secondary wave.)
S-wave

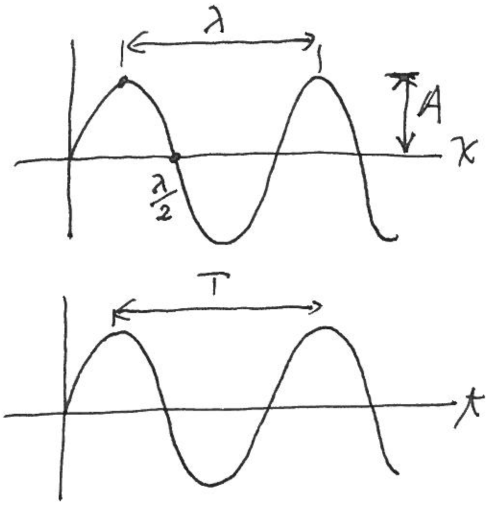
In a string



$$y(x,t) = y(x - vt, 0)$$
$$= f(x - vt) \rightarrow \text{traveling in } +\hat{x} \text{ direction}$$

$$y(x,t) = f(x + vt) \rightarrow \text{traveling in } -\hat{x} \text{ direction}$$

16.2 Sinusoidal waves



$f = \frac{1}{T}$

$y(x,0) = A \sin ax$

$A \equiv$ Amplitude

$a =$ constant to be determined

$y(0,0) = A \sin(a \cdot 0) = 0$

$y(\frac{\lambda}{2}, 0) = A \sin a(\frac{\lambda}{2}) = 0$

$$a \frac{\lambda}{2} = \pi \rightarrow a = \frac{2\pi}{\lambda}$$

$$\therefore \; y(x,0) = A \sin(\frac{2\pi}{\lambda} x)$$

$$\rightarrow y(x,t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

But $v = \frac{\lambda}{T}$

$$= A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

define $\frac{2\pi}{\lambda} \equiv k$ = (Angular) wave number

$\frac{2\pi}{T} \equiv \omega$ = Angular frequency

$$\rightarrow y(x,t) = A \sin(kx - \omega t)$$

$$= A \sin(kx - \omega t + \phi)$$

$v = \frac{\omega}{k}$ = Speed

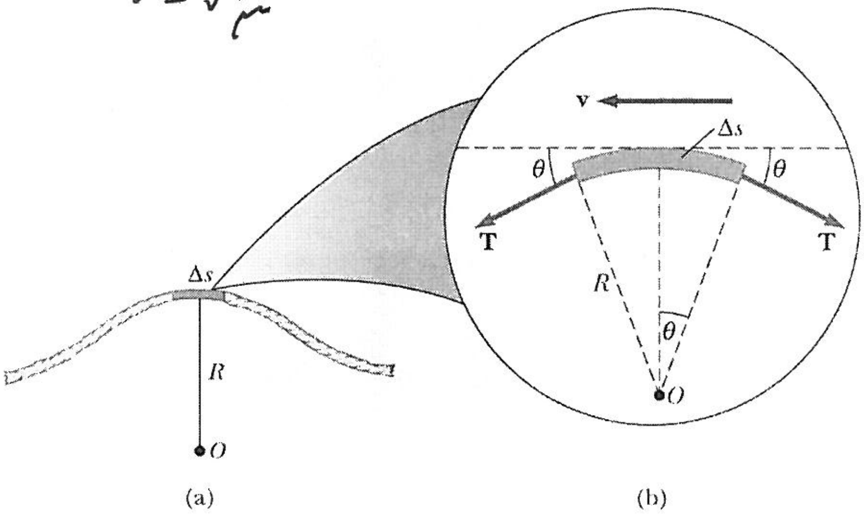
$v = \lambda f$

$$v_y = \left.\frac{dy}{dt}\right|_{x = \text{const.}} = \frac{\partial y}{\partial t} = -\omega A \cos(kx - \omega t), \quad v_{y,max} = \omega A$$

$$a_y = \left.\frac{dv_y}{dt}\right|_{x = \text{const}} = \frac{\partial v_y}{\partial t} = -\omega^2 A \sin(kx - \omega t), \quad a_{y,max} = \omega^2 A$$

16.3 The speed of waves on strings

$$v = \sqrt{\frac{T}{\mu}}$$



$$F_r = 2T\sin\theta \simeq 2T\theta$$

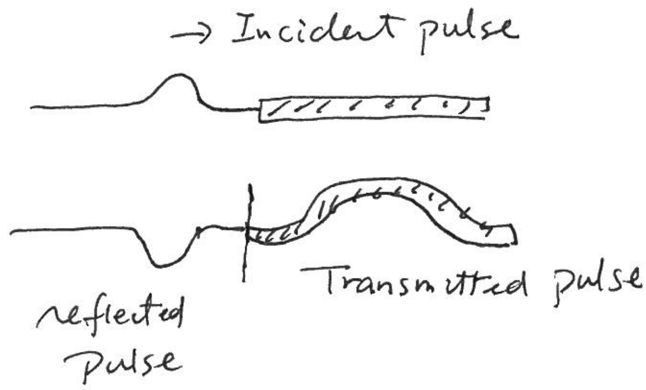
$$m = \mu \Delta s = \mu R 2\theta = 2\mu R\theta$$

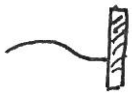
$$\text{But } F_r = ma = \frac{mv^2}{R} = 2T\theta$$

$$\therefore 2T\theta = \frac{mv^2}{R} = \frac{2\mu R\theta v^2}{R}$$

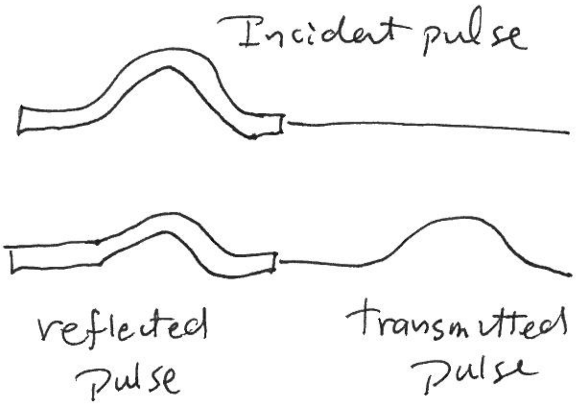
$$\rightarrow v = \sqrt{\frac{T}{\mu}}$$

16.4 Reflection and Transmission



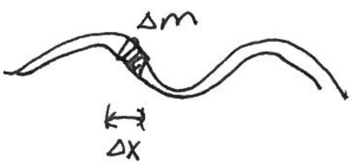


The inversion of the reflected pulse can be explained using Newton's 3rd law



Whether it will be inverted or not depends on the speed of the wave in each medium.

16.5 Rate of energy transfer by ~~Sinud~~ Sinusoidal waves in strings.



$$\Delta K = \frac{1}{2}(\Delta m) v_y^2$$

$$= \frac{1}{2}(\mu \Delta x) v_y^2 \rightarrow dK = \frac{1}{2}\mu\, dx\, v_y^2$$

$$= \frac{1}{2}\mu\left[\omega A \cos(kx-\omega t)\right]^2 dx$$

$$= \frac{1}{2}\mu \omega^2 A^2 \cos^2(kx-\omega t)\, dx$$

If $t=0$

$$dk = \frac{1}{2}\mu \omega^2 A^2 \cos^2(kx)\, dx$$

$$K_\lambda = \int dK = \int_0^\lambda \frac{1}{2}\mu \omega^2 A^2 \cos^2(kx)\, dx\ .$$

$$K_\lambda = \int_0^\lambda \frac{1}{2}\mu\omega^2 A^2 \cos^2(kx)\,dx$$

$$= \frac{1}{2}\mu\omega^2 A^2 \int_0^\lambda \cos^2 kx\,dx$$

$$= \frac{1}{2}\mu\omega^2 A^2 \int_0^\lambda \left(\frac{1-\cos 2kx}{2}\right) dx$$

$$= \frac{1}{2}\mu\omega^2 A^2 \left[\frac{1}{2}x - \frac{1}{4k}\underbrace{\sin 2kx}_{=0}\right]_0^\lambda$$

$$= \frac{1}{2}\mu\omega^2 A^2 \left(\frac{1}{2}\lambda\right)$$

$$= \frac{1}{4}\mu\omega^2 A^2 \lambda$$

Similarly the total potential energy in one wave length is

$$U_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$$

$$E_\lambda = K_\lambda + U_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

$P \equiv$ power of energy transfer

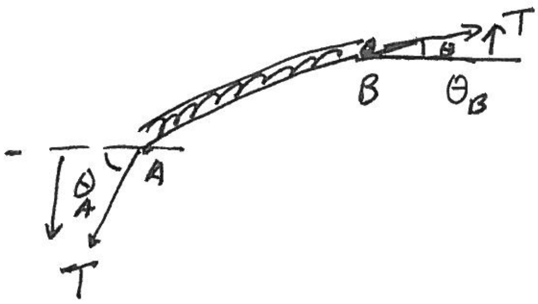
$$= \frac{\Delta E}{\Delta t} = \frac{E_\lambda}{T} = \frac{\frac{1}{2}\mu\omega^2 A^2 \lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 \left(\frac{\lambda}{T}\right)$$

$$\boxed{P = \frac{\Delta E}{\Delta t} = \frac{1}{2}\mu\omega^2 A^2 v}$$

16.6 Linear wave Equation

Wave equation

Net force in $\hat{y}$ direction



$$\sum F_y = T\sin\theta_B - T\sin\theta_A$$

$$= T(\sin\theta_B - \sin\theta_A) \quad \text{if } \theta \text{ is small, } \sin\theta \approx \tan\theta$$

$$\approx T(\tan\theta_B - \tan\theta_A)$$

$$= T\left(\frac{\partial y}{\partial x}\Big|_{\text{at } B} - \frac{\partial y}{\partial x}\Big|_A\right) = ma_y = \mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right)$$

$$\therefore \mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right) = T\left[\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A\right]$$

$$\text{or} \quad \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{(\partial y/\partial x)_B - (\frac{\partial y}{\partial x})_A}{\Delta x}$$

$$= \frac{\partial^2 y}{\partial x^2} \qquad \text{Note: } \frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\therefore \boxed{\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}} \quad \text{or} \quad \boxed{\frac{\mu}{T}\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}}$$

$$\Rightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 A\sin(kx - \omega t)$$

$$\frac{-\omega^2\mu}{T}\sin(kx - \omega t) = -k^2\sin(kx - \omega t)$$

$$\rightarrow k^2 = \frac{\mu}{T}\omega^2, \qquad v = \frac{\omega}{k}$$

$$\therefore v^2 = \frac{\omega^2}{k^2} = \frac{T}{\mu} \quad \rightarrow \quad \boxed{v = \sqrt{\frac{T}{\mu}}}$$