Homework problem #12 in §1.12 seems less straightforward. Here is a way to prove it.

To make clear what we seek to calculate, we need some definitions (they are similar to those used for proving Theorem 1.10.2).

$$
\begin{aligned}
B_1 &= A_1 A_2^c A_3^c \cdots A_n^c \\
B_{12} &= A_1 A_2 A_3^c \cdots A_n^c \\
\cdots &= \cdots \\
B_{123\cdots n} &= A_1^c A_2^c \cdots A_n^c.
\end{aligned}
$$

Other B sets can be similarly defined. We also abuse the notations by adopting the convention: Let $s \subset S = \{1, 2, \cdots, n\}$, we denote B_s as the event with all the elements in s as its subscript. For example, $s = \{1, 2, 3\}$ then $B_s = B_{123}$. Also $B_{\emptyset} = A_1^c A_2^c \cdots A_n^c$.

In the notations, #12 asks us to show that

$$P\left(\bigcup_{i=1}^{n} B_i\right) = \sum_{l=1}^{n} P($$

Proof. By Binomial Theorem,

$$\sum_{m=0}^{k} C_m^k(-1)^m = 0, \tag{3}$$

(noting that $(1-1)^k = \sum_{m=0}^{k} C_m^k(-1)^m 1^{k-m}$) and

$$(1+x)^k = \sum_{m=0}^{k} C_m^k x^m 1^{k-m}. \tag{4}$$

Differentiate both sides of (4) and then evaluate them at $x = -1$, we have

$$\sum_{m=1}^{k} mC_m^k(-1)^m = 0. \tag{5}$$

Thus by (3) and(5), thdofthirf(a)1(n-T)-1(sides)-3-equalesthefsthet