Note: The etude problem sets has 7 questions, for a total of 165 points. You should read/review your class notes first and then set yourself sometime to work on these problems. You may also treat it as a practice final. However, there are less problems in the real final exam (for 90 min exam time). For this etude problem sets, the working time is around 120 min. Remind you the **NENC** principle for my exam: explain your answer and write down necessary details of your calculation. No explanation/details = No credits. *Good Luck!*

1. *True or False?* In the following questions, determine whether the statement is true or false. Give an example if the statement is true; otherwise, give a counterexample.

(5) (a) If $E(X_i), Var(X_i) < \infty, i = 1, 2$ and X_1, X_2 are independent. Then $Var(3X_1 - 2X_2 + 3) = 9Var(X_1) + 4E(X_2)$.

(5) (b) If X, Y are independent random variable with finite expectations and variances then $Cov(X, Y) = 0$.

(5) (c) Let X, Y be two random variables with marginal pdf f_X, f_Y respectively. Then their joint pdf $f(x, y) = f_X(x)f_Y(y)$ for all x, y.

(5) (d) If X, Y are independent then $E(X/Y) = E(X)/E(Y)$ if all these expectations exist.

2. Let $X_1, \cdots, X_n$ be iid random variables from $N(\mu, \sigma^2)$

(10) (a) Show that $(X_1 - \mu)/\sigma$ is a standard normal random variable, i.e. $N(0, 1)$.

(10) (b) Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Show that $\bar{X} \sim N(\mu, \sigma^2/n)$.

3. Let X, Y have the joint pdf

$$f(x) = \begin{cases} c(1 + x + y) & \text{if } 0 < x < 1 \text{ and } 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

(10) (a) Determine c such that f defines a (joint) pdf

(15) (b) Compute $E(X^2 + XY - Y)$

(15) (c) Are X, Y independent? Are they uncorrelated?

4. Let X, Y have the joint pmf

$$f(x) = \begin{cases} cxy & \text{if } x = 1, 2 \text{ and } y = 1, 2, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

(10) (a) Determine c such that f defines a joint pmf

(15) (b) Compute $E(X^2 + XY - Y)$

(15) (c) Are X, Y independent? Are they uncorrelated?

5. Let X be a random variable with mgf

$$M_X(t) = \frac{1}{4}(e^{-t} + 2 + e^t).$$

(10) (a) Compute $E(2019\ X^{2019} + 2018\ X^{2018})$.

(10) (b) Let $Y = 3X - 1$. Compute $M_Y(t)$, the mgf of Y.

6. Let $X_1, \cdots, X_n \sim_{iid} N(2, 4)$.

(10) (a) Derive the joint pdf of (X_1, X_2). Recall if $X \sim N(\mu, \sigma^2)$ the the pdf of X is

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad \forall x \in (-\infty, \infty).$$

(10) (b) What is (the distribution of) $\sum_{k=1}^{n} (\frac{X_k - 2}{2})$?

(5) 7. Estimate the points you will get for the exam (excluding this problem).