

1. Under the setting of simple linear regression model, write down  $f$  explicitly and solve through risk minimization of

$$\begin{aligned} EPE(f) &= E(Y - f(X))^2 = \int (y - f(x))^2 dP_{Y,X}(y; x) \\ &= E_X E_{Y|X} [Y - f(X)]^2 | X : \end{aligned} \quad (1)$$

Compare the with usual LSE of and comment of their differences.

Do you think the question is ill-posed? Do you need extra assumptions/conditions to answer the question?

2. Follow §2.5 in HTF and use R, reproduce Fig 2.7 and Fig 2.9. You may need to install contributed R packages such as kkn.
3. Exercise 2.1, Exercise 2.5 and Exercise 2.6 in HTF.
4. Write down the algorithm for Succession Orthogonalization and iterative residual fitting respectively. Prove (or disprove) that the obtained  $\hat{\beta}$  also solves the normal equation.
5. Let  $\lambda$  be a positive number. The ridge estimate,  $\hat{\beta}_{ridge}$ , minimizes a regularized risk.

$$\hat{\beta}_{ridge} = \operatorname{argmin}_{\beta} (Y - X\beta)^t (Y - X\beta) + \lambda \beta^t \beta$$

Show that

$$\hat{\beta}_{ridge} = (X^t X + \lambda I)^{-1} X^t Y$$