# Class notes

1. Midterm will be held in class 1400-1600, 030618 (Wed) at B101. You can bring along a calculator and a cheatsheat of size A4 with you. I will explain this part in class. **Be Prepared!**.

2. Material covered in class (before and after midterm). Midterm Exam.

3. Outline of topics you should know for the final. Please refer to your notes for more details.

    1. Point Estmation: Joint probability density function, joint probability mass function and likelihood function. Maximum Likelihood Estimator. MLE for $X_1, \cdots, X_n$ iid from Bernoulli, Normal distribution.
    2. Confidence Intervals for Normal Mean: Calculate a $(1-\alpha)$ (exact) confidence interval for $\theta$ when $X_1, \cdots, X_n$ iid from $N(\theta, \sigma^2)$
        - Known $\sigma^2$: where $z_\alpha$ s.t. $\Phi(z_\alpha) = P(Z \le z_\alpha) = 1-\alpha$.

$$C_2(X) = [\bar{X} - \frac{\sigma}{\sqrt{n}} z_{\alpha/2}, \bar{X} - \frac{\sigma}{\sqrt{n}} z_{\alpha/2}],$$
$$C_{1L}(X) = [\bar{X} - \frac{\sigma}{\sqrt{n}} z_\alpha, \quad \infty),$$
$$C_{1U}(X) = (-\infty, \quad \bar{X} + \frac{\sigma}{\sqrt{n}} z_\alpha]$$

        - Unknown $\sigma^2$: where $t_{n-1;\alpha}$ s.t. $P(T_{n-1} \le t_{n-1;\alpha}) = 1-\alpha$.
          $S^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X})^2$

$$C_2(X) = [\bar{X} - \frac{S}{\sqrt{n}} t_{n-1;\alpha/2}, \bar{X} - \frac{S}{\sqrt{n}} t_{n-1;\alpha/2}],$$
$$C_{1L}(X) = [\bar{X} - \frac{S}{\sqrt{n}} t_{n-1;\alpha}, \quad \infty),$$
$$C_{1U}(X) = (-\infty, \quad \hat{X}026\frac{S}{\sqrt{n}} t_{n-1;\alpha}]$$

    3. Confidence interval for population mean (large sample): Calculate an asymptotic $(1-\alpha)$ (approximated) confidence interval for $\theta$ when $X_1, \cdots, X_n$ iid from a population and $E(X_i) = \theta$. $t_{n-1;\alpha}$ s.t. $P(T_{n-1} \le t_{n-1;\alpha}) = 1-\alpha$.
       $S^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X})^2$

$$C_2(X) = [\bar{X} - \frac{S}{\sqrt{n}} t_{n-1;\alpha/2}, \bar{X} - \frac{S}{\sqrt{n}} t_{n-1;\alpha/2}],$$
$$C_{1L}(X) = [\bar{X} - \frac{S}{\sqrt{n}} t_{n-1;\alpha}, \quad \infty),$$
$$C_{1U}(X) = (-\infty, \quad \bar{X} + \frac{S}{\sqrt{n}} t_{n-1;\alpha}]$$

Note: since $n$ is assumed to be large, say greater than 30 here. $t_{n-1;\alpha} \approx z_\alpha$. It makes no practical differences.

4. Hypothesis Testing: Setting hypotheses (which should be null, alternative?)
5. Assume $\theta = EX_i$, the common population mean. Typical hypothesis testing problems are

$$H_0 : \theta \leq \theta_0 \quad vs. \quad H_1 : \theta > \theta_0 \tag{1}$$
$$H_0 : \theta \geq \theta_0 \quad vs. \quad H_1 : \theta < \theta_0 \tag{2}$$
$$H_0 : \theta = \theta_0 \quad vs. \quad H_1 : \theta \neq \theta_0 \tag{3}$$

6. Hypothesis testing problems for normal mean. Assume $X_1, \cdots, X_n$ iid $N(\theta, \sigma^2)$ where $\sigma^2$ is known.
7. Hypothesis testing problems for normal mean. Assume $X_1, \cdots, X_n$ iid $N(\theta, \sigma^2)$ where $\sigma^2$ is unknown.
8. Connection hypothesis test, confidence interval and p-value.
   For testing (3), when observe $\mathbf{X} = \mathbf{x}$

$$\text{Reject} \quad H_0 \quad \text{at level} \quad \alpha \leftrightarrow \quad \text{p-value} \quad < \alpha \leftrightarrow \quad \theta_0 \notin C(\mathbf{x})$$

   where $C(\mathbf{X})$ is a two-sided $(1-\alpha)$ confidence interval for $\theta$.
9. Hypothesis testing problems for two normal means:
   Let $X_1, \cdots, X_n \sim N(\theta_X, \sigma_X^2)$ and $Y_1, \cdots, Y_m \sim N(\theta_Y, \sigma_Y^2)$. Desire to test

$$H_0 : \theta_X - \theta_Y = 0 \quad vs. \quad H_1 : \theta_X - \theta_Y \neq 0$$

   - Design: Matched pairs or independent samples
   - Equal Variances?: $\sigma_X^2 = \sigma_Y^2$ or $\sigma_X^2 \neq \sigma_Y^2$?
10. Checking normality assumption using descriptive statistics and graphs.
11. Interpreting outputs from `R` and derive, say, confidence interval and decide to reject or not reject null hypothesis.