榮譽第一

國立東華大學 應用數學系 ＿＿學年度第＿＿學期 考試科目：＿＿＿＿ □期中□期末考試試卷

sol. to Quiz 4. 2014 12 31

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#1. The volume of the solid

$= \int_1^{e^2} \pi \left(\frac{1}{\sqrt{x}}\right)^2 dx = \pi \int_1^{e^2} \frac{1}{x}\, dx$

$= \pi \ln x \Big|_1^{e^2} = \pi \ln e^2 - \pi \ln 1 = 2\pi$

#2.

(a) $\int_0^1 \frac{x}{e^{2x}}\, dx = \int_0^1 x e^{-2x}\, dx$

$= \int_0^1 -\frac{1}{2} x \; d e^{-2x}$

$= -\frac{1}{2} x e^{-2x} \Big|_0^1 + \frac{1}{2}\int_0^1 e^{-2x}\, dx$

$= -\frac{1}{2} e^{-2} + \left(-\frac{1}{4} e^{-2x}\right)\Big|_0^1$

$= -\frac{1}{2} e^{-2} - \frac{1}{4} e^{-2} + \frac{1}{4} = \frac{1}{4} - \frac{3}{4} e^{-2}$

(b) $\int_1^{e^2} x \ln \sqrt[3]{x} = \int_1^{e^2} \frac{1}{3} x \ln x\, dx$

$= \int_1^{e^2} \frac{1}{6} \ln x \; d x^2$

$= \frac{1}{6} x^2 \ln x \Big|_1^{e^2} - \int_1^{e^2} x^2 \cdot \frac{1}{6} \frac{1}{x}\, dx$

$= \frac{1}{3} e^4 - \frac{1}{6} \int_1^{e^2} x\, dx$

$= \frac{1}{3} e^4 - \left(\frac{1}{12} x^2 \Big|_1^{e^2}\right)$

$= \frac{1}{3} e^4 - \frac{1}{12} e^4 + \frac{1}{12} = \frac{1}{4} e^4 + \frac{1}{12}$

(c) $\int \frac{\ln x}{x^2}\, dx = \int \ln x \cdot x^{-2}\, dx$

$= \int -\ln x \;\; d x^{-1}$

$= -\frac{1}{x} \ln x + \int x^{-1} \frac{1}{x}\, dx$

$= -\frac{\ln x}{x} + \int x^{-2}\, dx$

$= -\frac{\ln x}{x} - \frac{1}{x} + C$

(d) $\int x^3 e^{x^2} dx = \int \frac{1}{2} x^2 \, d e^{x^2}$

$= \frac{1}{2} x^2 e^{x^2} - \int e^{x^2} \cdot x\, dx$

$= \frac{1}{2} x^2 e^{x^2} - \left[\int \frac{1}{2} \, d e^{x^2} \right]$

$= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C$

(e) $\int \frac{1}{16 - 3u^2}\, du$

$= \int \frac{1}{(4-\sqrt{3}u)(4+\sqrt{3}u)}\, du$

$= \int \frac{1}{8}\left(\frac{1}{4-\sqrt{3}u} + \frac{1}{4+\sqrt{3}u} \right) du$

$= \frac{1}{8} \int \frac{1}{4-\sqrt{3}u}\, du + \frac{1}{8} \int \frac{1}{4+\sqrt{3}u}\, du$

$= \frac{-1}{8\sqrt{3}} \int \frac{1}{4-\sqrt{3}u}\, d(4-\sqrt{3}u)$

$+ \frac{1}{8\sqrt{3}} \int \frac{1}{4+\sqrt{3}u}\, d(4+\sqrt{3}u)$

$= \frac{1}{8\sqrt{3}} \ln|4+\sqrt{3}u| - \frac{1}{8\sqrt{3}} \ln|4-\sqrt{3}u| + C$

$= \frac{1}{8\sqrt{3}} \ln\left|\frac{4+\sqrt{3}u}{4-\sqrt{3}u}\right| + C$

(f) $\int (\ln x)^3\, dx$

$= x(\ln x)^3 - \int x \cdot 3(\ln x)^2 \cdot \frac{1}{x}\, dx$

$= x(\ln x)^3 - \int 3(\ln x)^2\, dx$

$= x(\ln x)^3 - \left[3x(\ln x)^2 - \int x \cdot 6(\ln x)\frac{1}{x}\, dx \right]$

$= x(\ln x)^3 - 3x(\ln x)^2 + \int 6 \ln x\, dx$

$= x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x$

$- \int x \cdot 6 \cdot \frac{1}{x}\, dx$

$= x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x$

$- 6x + C$

(g) $\int x \cos(2x)\, dx$

$= \int \frac{1}{2} x \; d\sin(2x)$

$= \frac{1}{2} x \sin(2x) - \int \sin(2x) \frac{1}{2}\, dx$

$= \frac{1}{2} x \sin(2x) + \int \frac{1}{4} \; d\cos(2x)$

$= \frac{1}{2} x \sin(2x) + \frac{1}{4}\cos(2x) + C.$

#3.

$$f(x) = \int (x+1)e^{-x}dx$$

$$= \int -(x+1)\, de^{-x}$$

$$= -(x+1)e^{-x} + \int e^{-x}dx$$

$$= -(x+1)e^{-x} - e^{-x} + c.$$

$$f(1) = -2e^{-1} - e^{-1} + c = 5$$

$$\Rightarrow \quad c = 5 + 3e^{-1}$$

$$\therefore f(x) = -(x+1)e^{-x} - e^{-x} + 5 + 3e^{-1}$$

#4. $\frac{dx}{dt} = x^2 \cos t.$

$$\Leftrightarrow \quad \int \frac{1}{x^2}dx = \int \cos t\, dt.$$

i.e. $-\frac{1}{x} = \sin t + c.$

$$\Rightarrow \quad x(t) = \frac{-1}{\sin t + c}$$

$x(\pi) = 1 \quad \Rightarrow \quad -1 = \sin(\pi) + c$

$$\Rightarrow \quad c = -1$$

$$\Rightarrow x(t) = \frac{-1}{\sin(t) - 1}.$$