榮譽第一

國立東華大學應用數學系 ___學年度第___學期 考試科目：108 □期中□期末考試試卷 P.1

學號 411211314 姓名 王顯綺 系所 應數三 班別 ___ 任課教師：___ 共___張

10 1. Let $X \sim f(x;\theta)$, $\theta_0 \ne \theta_1$, $\theta_0,\theta_1 \in \Omega$, and given $\alpha \in (0,1)$

For testing $H_0:\theta=\theta_0$ v.s. $H_1:\theta=\theta_1$

(i) (Existence)

$\exists$ a test $\phi(x)$ and constant $c>0$

s.t. (a) $E_{\theta_0}\phi(X)=\alpha$

(b) $\phi(x)=\begin{cases}1, & \text{if } f(x;\theta_1) > c f(x;\theta_0)\\ 0, & \text{if } f(x;\theta_1) < c f(x;\theta_0)\end{cases}$

(ii) (Sufficient condition)

If a test ϕ^* satisfies (a) & (b) in (i)

for some $c>0$, then ϕ^* is MP test

within test whose level $\le \alpha$

i.e. $\forall \phi$ w/ $E_{\theta_0}\phi(X) \le \alpha$, $E_{\theta_1}\phi^*(X) \ge E_{\theta_1}\phi(X)$

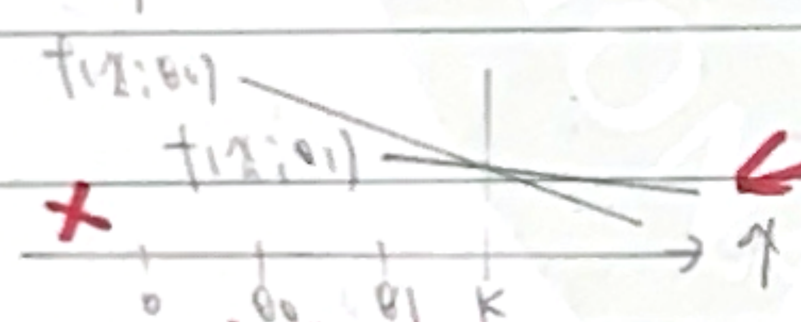
(iii) (Necessary condition)

If ϕ^* is a MP level α test

then ϕ^* satisfies (a) & (b) in (i) for some $c>0$

9 2. (a) $X_1,\dots,X_n$ iid $\frac{\theta}{x^2}$, $\theta \le x < \infty$, $0<\theta_1<\theta_0$, $\alpha \in (0,1)$

To test $H_0:\theta=\theta_0$ v.s. $H_1:\theta=\theta_1$



$\xrightarrow{\text{N-P lemma}}$ $\phi(x)=\begin{cases}1, & \text{if } f(x;\theta_1) > c f(x;\theta_0) \ \oplus\\ \gamma, & =\\ 0, & <\end{cases}$

w/ $c>0$, $\gamma \in [0,1]$ s.t. $\alpha = E_{\theta_0}\phi(X)$

$\oplus \iff X_{(1)} > k$，由圖知

$\alpha = E_{\theta_0}\phi(X) = P_{\theta_0}(X_{(1)}>k) = [P_{\theta_0}(X>k)]^n$ take $\gamma=0$

$= [1-P_{\theta_0}(X \le k)]^n = \left[1-\int_{\theta_0}^{k}\frac{\theta_0}{x^2}dx\right]^n$

$= \left[1+\frac{\theta_0}{x}\Big|_{\theta_0}^{k}\right]^n = \left[1+\frac{\theta_0}{k}-1\right]^n = \left(\frac{\theta_0}{k}\right)^n$

$\Rightarrow \frac{\theta_0}{k} = \alpha^{\frac{1}{n}} \Rightarrow k = \frac{\theta_0}{\sqrt[n]{\alpha}}$

i.e. MP level α test rej H_0 iff $X_{(1)} > \frac{\theta_0}{\sqrt[n]{\alpha}}$

The power of ϕ

$= E_{\theta_1}\phi(X) = P_{\theta_1}\left(X_{(1)} > \frac{\theta_0}{\sqrt[n]{\alpha}}\right) = \left[P_{\theta_1}\left(X > \frac{\theta_0}{\sqrt[n]{\alpha}}\right)\right]^n$

$= \left[1-P_{\theta_1}\left(X \le \frac{\theta_0}{\sqrt[n]{\alpha}}\right)\right]^n = \left[1-\int_{\theta_1}^{\theta_0/\sqrt[n]{\alpha}}\frac{\theta_1}{x^2}dx\right]^n$

$= \left[1+\frac{\theta_1}{x}\Big|_{\theta_1}^{\theta_0/\sqrt[n]{\alpha}}\right]^n = \left[1+\frac{\theta_1}{\theta_0/\sqrt[n]{\alpha}}-1\right]^n = \left(\frac{\theta_1}{\frac{\theta_0}{\sqrt[n]{\alpha}}}\right)^n$

$= \alpha\left(\frac{\theta_1}{\theta_0}\right)^n$

9 (b) $\forall \theta_2 > \theta_1$,

$\frac{f(x;\theta_2)}{f(x;\theta_1)} = \frac{(\theta_2)^n \prod \frac{1}{x_i^2} I(\theta_2 \le x_i < \infty)}{(\theta_1)^n \prod \frac{1}{x_i^2} I(\theta_1 \le x_i < \infty)}$

$= \left(\frac{\theta_2}{\theta_1}\right)^n \frac{I(X_{(1)} \ge \theta_2)}{I(X_{(1)} \ge \theta_1)}$ 0 θ1 θ2

$= \begin{cases}\left(\frac{\theta_2}{\theta_1}\right)^n, & \text{if } X_{(1)} \ge \theta_2\\ 0, & \text{if } \theta_1 \le X_{(1)} < \theta_2\end{cases}$

$\Rightarrow \frac{f(x;\theta_2)}{f(x;\theta_1)} \nearrow$ in $X_{(1)}$

$\Rightarrow f$ has MLR in $X_{(1)}$

UMP level α test for $H_0:\theta \ge \theta_0$ v.s. $H_1:\theta<\theta_0$

is $\phi(x)=\begin{cases}1, & \text{if } X_{(1)} > k\\ \gamma, & =\\ 0, & <\end{cases}$

w/ k, $\gamma \in [0,1]$ s.t. $\alpha = E_{\theta_0}\phi(X)$

$\alpha = E_{\theta_0}\phi(X) = P_{\theta_0}(X_{(1)}>k) = [P_{\theta_0}(X>k)]^n$, take $\gamma=0$

$= [1-P_{\theta_0}(X \le k)]^n = \left[1-\int_{\theta_0}^{k}\frac{\theta_0}{x^2}dx\right]^n$

$= \left[1+\frac{\theta_0}{x}\Big|_{\theta_0}^{k}\right]^n = \left[1+\frac{\theta_0}{k}-1\right]^n = \left(\frac{\theta_0}{k}\right)^n$

$\Rightarrow \frac{\theta_0}{k} = \alpha^{\frac{1}{n}} \Rightarrow k = \frac{\theta_0}{\sqrt[n]{\alpha}}$

i.e. UMP level α test rej H_0 iff $X_{(1)} > \frac{\theta_0}{\sqrt[n]{\alpha}}$

$\Rightarrow$ (b)'s test is same as (a)'s test

10 3. $\exists$ a UMP level α test $\phi^*(x)$,

and $\phi_\alpha = \alpha$, $E_\theta \phi_\alpha(X) = \alpha$

$\because \phi^*(x)$ is UMP

$\therefore E_\theta \phi^*(X) \ge E_\theta \phi_\alpha(X) = \alpha$

$\Rightarrow \phi^*$ is unbiased

i.e. ϕ^* is UMPU level α test

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4. (10) $X_1,\dots,X_n$ iid $N(\theta,\sigma_0^2)$, $\sigma_0^2>0$

To test $H_0:\theta\ge\theta_0$ v.s. $H_1:\theta<\theta_0$

under $\omega_0=\{\theta:\theta\ge\theta_0\}$

$$\hat\theta_{\omega_0}=\begin{cases}\bar x, & \text{if } \bar x\ge\theta_0\\ \theta_0, & \text{if } \bar x<\theta_0\end{cases}$$

under $\Omega=\{\theta:\theta\in R\}$

$\hat\theta_\Omega=\bar x$

$$\lambda(x)=\frac{L(\hat\theta_{\omega_0};x)}{L(\hat\theta_\Omega;x)}=\frac{\left(\frac{1}{\sqrt{2\pi}\sigma_0}\right)^n\exp\left(-\frac{\sum(x_i-\hat\theta_{\omega_0})^2}{2\sigma_0^2}\right)}{\left(\frac{1}{\sqrt{2\pi}\sigma_0}\right)^n\exp\left(-\frac{\sum(x_i-\hat\theta_\Omega)^2}{2\sigma_0^2}\right)}$$

$$=\begin{cases}1, & \text{if } \bar x\ge\theta_0\\ \dfrac{\exp\left(-\frac{\sum(x_i-\theta_0)^2}{2\sigma_0^2}\right)}{\exp\left(-\frac{\sum(x_i-\bar x)^2}{2\sigma_0^2}\right)}, & \text{if } \bar x<\theta_0\end{cases}$$

$<k$ w/ $k\in(0,1)$

$$\Leftrightarrow \frac{\exp\left(-\frac{\sum x_i^2-2\theta_0\sum x_i+n\theta_0^2}{2\sigma_0^2}\right)}{\exp\left(-\frac{\sum x_i^2-2\bar x\sum x_i+n\bar x^2}{2\sigma_0^2}\right)}<k,\ \bar x<\theta_0$$

$$\Leftrightarrow \exp\left(\frac{-\sum x_i^2+2\theta_0 n\bar x-n\theta_0^2+\sum x_i^2-2n\bar x^2+n\bar x^2}{2\sigma_0^2}\right)<k,\ \bar x<\theta_0$$

$$\Leftrightarrow \exp\left(\frac{-n\bar x^2+2n\theta_0\bar x-n\theta_0^2}{2\sigma_0^2}\right)<k,\ \bar x<\theta_0$$

$$\Leftrightarrow \frac{-n(\bar x^2-2\theta_0\bar x+\theta_0^2)}{2\sigma_0^2}<k',\ \bar x<\theta_0$$

$$\Leftrightarrow (\bar x-\theta_0)^2>k'',\ \bar x<\theta_0$$

$\Leftrightarrow \bar x<c$

To have level α

$$\alpha=P_{\theta_0}(\bar x<c)=P_{\theta_0}\Bigg(\underbrace{\frac{\sqrt n(\bar x-\theta_0)}{\sigma_0}}_{N(0,1)}<\frac{\sqrt n(c-\theta_0)}{\sigma_0}\Bigg)$$

$z_{1-\alpha} = -z_\alpha$

$$\Rightarrow \frac{\sqrt n(c-\theta_0)}{\sigma_0}=z_{1-\alpha}\Rightarrow c=\theta_0+\frac{\sigma_0}{\sqrt n}z_{1-\alpha}$$

i.e. a level α LRT rej H_0 iff $\bar x<\theta_0+\frac{\sigma_0}{\sqrt n}z_{1-\alpha}$

$$f(x;\theta)=\left(\frac{1}{\sqrt{2\pi}\sigma_0}\right)^n\exp\left(-\frac{\sum(x_i-\theta)^2}{2\sigma_0^2}\right)$$

$$=\left(\frac{1}{\sqrt{2\pi}\sigma_0}\right)^n\exp\left(-\frac{\sum x_i^2}{2\sigma_0^2}\right)\exp\left(+\frac{2\theta\sum x_i}{2\sigma_0^2}\right)\exp\left(-\frac{n\theta^2}{2\sigma_0^2}\right)$$

$$=\underbrace{\left(\frac{1}{\sqrt{2\pi}\sigma_0}\right)^n\exp\left(-\frac{n\theta^2}{2\sigma_0^2}\right)}_{c(\theta)}\exp\Big(\underbrace{\frac{n\theta}{\sigma_0^2}}_{b(\theta)}\underbrace{\bar x}_{T(x)}\Big)\underbrace{\exp\left(-\frac{\sum x_i^2}{2\sigma_0^2}\right)}_{h(x)}$$

w/ $b(\theta)=\frac{n\theta}{\sigma_0^2}\uparrow$ in θ

$\Rightarrow f$ has MLR in $\bar x$

UMP level α test for $H_0:\theta\ge\theta_0$ v.s. $H_1:\theta<\theta_0$

$$\text{is } \phi(x)=\begin{cases}1, & \text{if } \bar x<k\\ \gamma & =\\ 0 & >\end{cases}$$

w/ $k,\gamma\in[0,1]$ s.t. $\alpha=E_{\theta_0}\phi(X)$

$$\alpha=E_{\theta_0}\phi(X)=P_{\theta_0}(\bar x<k)=P_{\theta_0}\Bigg(\underbrace{\frac{\sqrt n(\bar x-\theta)}{\sigma_0}}_{N(0,1)}<\frac{\sqrt n(k-\theta_0)}{\sigma_0}\Bigg),\ \text{take } \gamma=0$$

$$\Rightarrow \frac{\sqrt n(k-\theta_0)}{\sigma_0}=z_{1-\alpha}\Rightarrow k=\theta_0+\frac{\sigma_0}{\sqrt n}z_{1-\alpha}$$

i.e. UMP level α test rej H_0 iff $\bar x<\theta_0+\frac{\sigma_0}{\sqrt n}z_{1-\alpha}$

Hence, LRT is UMP

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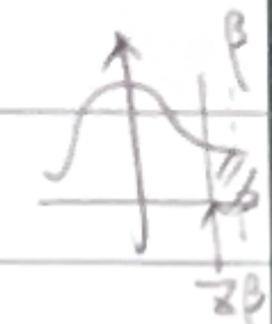
(b) $P_\theta\left(\bar x\ge\theta_0+\frac{\sigma_0}{\sqrt n}z_{1-\alpha}\right)\le\beta$



$$=P_\theta\left(\frac{\sqrt n(\bar x-\theta)}{\sigma_0}\ge\frac{\sqrt n(\theta_0-\theta+\frac{\sigma_0}{\sqrt n}z_{1-\alpha})}{\sigma_0}\right)\le\beta$$

$\Rightarrow \frac{\sqrt n}{\sigma_0}(\theta_0-\theta)+z_{1-\alpha}\ge z_\beta$ w/ $\alpha=0.05$, $\beta=0.2$, $\theta=\theta_0-0.01\sigma_0$

$\frac{\sqrt n}{\sigma_0}(\theta_0-\theta_0+0.01\sigma_0)\ge z_{0.2}-z_{0.95}$

$0.01\sqrt n\ge 0.845-(-1.645)=2.49$

$\sqrt n\ge 249$

$n\ge 62001$

$\Rightarrow$ Take $n=62001$

5. (10) $X_1,\dots,X_n$ iid $\theta(1-x)^{\theta-1}$, $x\in(0,1)$, $\theta>0$, θ = unknown

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$$f(x;\theta)=\theta^n\prod_{i=1}^n(1-x_i)^{\theta-1}=\theta^n\prod_{i=1}^n\exp[(\theta-1)\log(1-x_i)]$$

$$=\underbrace{\theta^n}_{c(\theta)}\exp[\underbrace{(\theta-1)}_{b(\theta)}\underbrace{\textstyle\sum\log(1-x_i)}_{T(x)}]\cdot\underbrace{1}_{h(x)}$$

w/ $b(\theta)=\theta-1\uparrow$ in θ

$\Rightarrow f$ has MLR in $\sum\log(1-x_i)$

note: Let $Y=-\log(1-X)$, then $X=1-e^{-Y}$

$f_Y(y)=f_X(1-e^{-y})\,|e^{-y}|$

$=\theta(1-(1-e^{-y}))^{\theta-1}\cdot e^{-y}=\theta e^{-y(\theta-1)}e^{-y}$

$=\theta e^{-\theta y}$, $\theta>0$, $y>0$

$Y\sim Gamma(1,\frac{1}{\theta})$, $\underbrace{\sum Y_i}_{W}\sim Gamma(n,\frac{1}{\theta})$

$f_W(w)=\frac{1}{\Gamma(n)(\frac{1}{\theta})^n}w^{n-1}\exp\left(-\frac{w}{\frac{1}{\theta}}\right)$ $=\frac{m}{2}\Rightarrow m=2\theta w$

$2\theta\sum Y_i=-2\theta\sum\log(1-X)\sim Gamma(n,2)=\chi^2_{2n}$

UMP level α test for $H_0:\theta\le\theta_0$ v.s. $H_1:\theta>\theta_0$

$$\text{is } \phi(x)=\begin{cases}1, & \text{if } \sum\log(1-x_i)>k\\ \gamma & =\\ 0 & <\end{cases}$$

w/ $k,\gamma\in[0,1]$ s.t. $\alpha=E_{\theta_0}\phi(X)$

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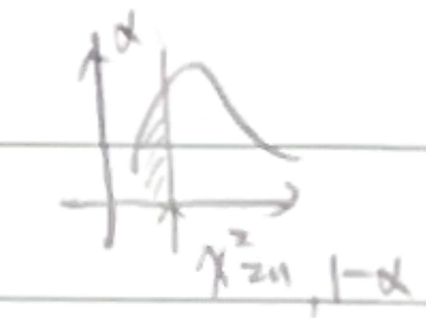
學號 411211314 姓名 王詠晴 系所 應數三 班別 ______ 任課教師：__________ 共____張

$\alpha = E_{\theta_0}\phi(\underline{X}) = P_{\theta_0}\left(\sum \log(1-X_i) > k\right)$, take $\gamma = 0$

$= P_{\theta_0}\left(\underbrace{-2\theta_0 \sum \log(1-X_i)}_{\sim \chi^2_{2n}} < -2\theta_0 k\right)$



$\Rightarrow -2\theta_0 k = \chi^2_{2n,1-\alpha} \Rightarrow k = -\frac{1}{2\theta_0}\chi^2_{2n,1-\alpha}$

i.e. UMP level α test rej H_0

iff $\sum \log(1-X_i) > -\frac{1}{2\theta_0}\chi^2_{2n,1-\alpha}$

10 (b) The power of $\phi = E_\theta \phi(\underline{X}) = P_\theta\left(\sum \log(1-X_i) > k\right)$

$= P_\theta\left(-2\theta \sum \log(1-X_i) < -2\theta\left(-\frac{1}{2\theta_0}\chi^2_{2n,1-\alpha}\right)\right) \geq \beta$

$\Rightarrow \frac{\theta}{\theta_0}\chi^2_{2n,1-\alpha} \geq \chi^2_{2n,1-\beta}$ w/ $\alpha = 0.05$, $\theta_1 = 1000 = \theta$, $\beta = 0.9$, $\theta_0 = 250$

$4\chi^2_{2n,0.95} \geq \chi^2_{2n,0.1}$ (*)

$\Rightarrow$ Take n be the smallest integer s.t. (*) holds

20 6. $X_1, \dots, X_n$ iid $U(0,\theta)$, $\theta > 0$

$\forall \theta_2 > \theta_1$, $\frac{f(\underline{x};\theta_2)}{f(\underline{x};\theta_1)} = \frac{\left(\frac{1}{\theta_2}\right)^n I(0 \leq x_i \leq \theta_2)}{\left(\frac{1}{\theta_1}\right)^n I(0 \leq x_i \leq \theta_1)}$

$= \left(\frac{\theta_1}{\theta_2}\right)^n \frac{I(x_{(n)} \leq \theta_2)}{I(x_{(n)} \leq \theta_1)}$

$= \begin{cases} \left(\frac{\theta_1}{\theta_2}\right)^n, & \text{if } 0 \leq x_{(n)} \leq \theta_1 \\ \infty, & \text{if } x_{(n)} > \theta_1 \end{cases}$

$\Rightarrow \frac{f(\underline{x};\theta_2)}{f(\underline{x};\theta_1)} \nearrow$ in $x_{(n)}$

$\Rightarrow f$ has MLR in $x_{(n)}$

To test $H_0: \theta = \theta_0$ v.s. $H_1: \theta \neq \theta_0$

$\phi(\underline{x}) = \begin{cases} 1, & \text{if } x_{(n)} > \theta_0 \text{ or } x_{(n)} < k \\ 0 & \text{o.w.} \end{cases}$

w/ k, s.t. $\alpha = E_{\theta_0}\phi(\underline{X})$

$\alpha = E_{\theta_0}\phi(\underline{X}) = \underbrace{P_{\theta_0}(X_{(n)} > \theta_0)}_{0} + P_{\theta_0}(X_{(n)} < k)$

$= [P_{\theta_0}(X < k)]^n = \left[\int_0^k \frac{1}{\theta_0}dx\right]^n = \left(\frac{k}{\theta_0}\right)^n$

$\Rightarrow \alpha^{\frac{1}{n}} = \frac{k}{\theta_0} \Rightarrow k = \theta_0 \alpha^{\frac{1}{n}}$

i.e. $\phi(\underline{x}) = \begin{cases} 1, & \text{if } x_{(n)} > \theta_0 \text{ or } x_{(n)} < \theta_0 \alpha^{\frac{1}{n}} \\ 0 & \text{o.w.} \end{cases}$

The power of $\phi = E_\theta \phi(\underline{X}) = \begin{cases} E_\theta \phi(\underline{X}), & \theta \geq \theta_0 \\ E_\theta \phi(\underline{X}), & \theta < \theta_0 \end{cases}$

Case 1. $\theta \geq \theta_0$

$E_\theta \phi(\underline{X}) = P_\theta(X_{(n)} > \theta_0) + P_\theta(X_{(n)} < \theta_0 \alpha^{\frac{1}{n}})$

$= 1 - P_\theta(X_{(n)} \leq \theta_0) + \left[P_\theta(X < \theta_0 \alpha^{\frac{1}{n}})\right]^n$

$= 1 - [P_\theta(X \leq \theta_0)]^n + \left[\int_0^{\theta_0\alpha^{\frac{1}{n}}} \frac{1}{\theta}dx\right]^n$

$= 1 - \left[\int_0^{\theta_0} \frac{1}{\theta}dx\right]^n + \left(\frac{\theta_0 \alpha^{\frac{1}{n}}}{\theta}\right)^n$

$= 1 - \left(\frac{\theta_0}{\theta}\right)^n + \alpha\left(\frac{\theta_0}{\theta}\right)^n$

$= 1 - (1-\alpha)\left(\frac{\theta_0}{\theta}\right)^n$

Case 2. $\theta < \theta_0$

$E_\theta \phi(\underline{X}) = \underbrace{P_\theta(X_{(n)} > \theta_0)}_{0} + P_\theta(X_{(n)} < \theta_0 \alpha^{\frac{1}{n}})$

$= \left[P_\theta(X < \theta_0 \alpha^{\frac{1}{n}})\right]^n$

$= \left[\int_0^{\theta_0\alpha^{\frac{1}{n}}} \frac{1}{\theta}dx\right]^n$

$= \left(\frac{\theta_0 \alpha^{\frac{1}{n}}}{\theta}\right)^n$

$= \alpha\left(\frac{\theta_0}{\theta}\right)^n$

UMP level α test for $H_0': \theta \leq \theta_0$ v.s. $H_1': \theta > \theta_0$

is $\phi_1(\underline{x}) = \begin{cases} 1, & \text{if } x_{(n)} > k \\ \gamma & = \\ 0 & < \end{cases}$

w/ $k, \gamma \in [0,1]$ s.t. $\alpha = E_{\theta_0}\phi(\underline{X})$

$\alpha = E_{\theta_0}\phi_1(\underline{X}) = P_{\theta_0}(X_{(n)} > k) = 1 - P_{\theta_0}(X_{(n)} \leq k)$

$= 1 - [P_{\theta_0}(X \leq k)]^n = 1 - \left[\int_0^k \frac{1}{\theta_0}dx\right]^n$

$= 1 - \left(\frac{k}{\theta_0}\right)^n$

$\Rightarrow \left(\frac{k}{\theta_0}\right)^n = 1-\alpha \Rightarrow k = \theta_0(1-\alpha)^{\frac{1}{n}}$

i.e. UMP level α test rej H_0 iff $X_{(n)} > \theta_0(1-\alpha)^{\frac{1}{n}}$

The power of $\phi_1 = E_\theta\phi_1(\underline{X}) = P_\theta(X_{(n)} > \theta_0(1-\alpha)^{\frac{1}{n}})$

$= 1 - P_\theta(X_{(n)} \leq \theta_0(1-\alpha)^{\frac{1}{n}}) = 1 - \left[P_\theta(X \leq \theta_0(1-\alpha)^{\frac{1}{n}})\right]^n$

$= 1 - \left[\int_0^{\theta_0(1-\alpha)^{\frac{1}{n}}} \frac{1}{\theta}dx\right]^n = 1 - \left(\frac{\theta_0(1-\alpha)^{\frac{1}{n}}}{\theta}\right)^n$

$= 1 - (1-\alpha)\left(\frac{\theta_0}{\theta}\right)^n$, $\because \phi_1$ is UMP

$\geq E_\theta \phi^*(\underline{X})$, $\forall \phi^*$ is a level α test for H_1'

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UMP level α test for $H_0^2: \theta \ge \theta_0$ vs. $H_1^2: \theta < \theta_0$

is $\phi_2(X) = \begin{cases} 1, & \text{if } X_{(n)} < k \\ \gamma & = \\ 0 & > \end{cases}$

w/ $k, \gamma \in [0,1]$ s.t. $\alpha = E_{\theta_0}\phi(X)$

$\alpha = E_{\theta_0}\phi_2(X) = P_{\theta_0}(X_{(n)} < k) = [P_{\theta_0}(X<k)]^n$

$= [\int_0^k \frac{1}{\theta_0} dx]^n = (\frac{k}{\theta_0})^n$

$\Rightarrow k = \theta_0 \alpha^{\frac{1}{n}}$

i.e., UMP level α test rej H_0 iff $X_{(n)} < \theta_0 \alpha^{\frac{1}{n}}$

The power of $\phi_2 = E_\theta \phi_2(X) = P_\theta(X_{(n)} < \theta_0 \alpha^{\frac{1}{n}})$

$= [P_\theta(X < \theta_0 \alpha^{\frac{1}{n}})]^n = [\int_0^{\theta_0 \alpha^{\frac{1}{n}}} \frac{1}{\theta} dx]^n$

$= (\frac{\theta_0 \alpha^{\frac{1}{n}}}{\theta})^n = \alpha(\frac{\theta_0}{\theta})^n$, $\because \phi_2$ is UMP

$\ge E_\theta \phi^*(X)$, $\forall \phi^*$ is a level α test for H^2

$\Rightarrow E_\theta \phi(X) = \begin{cases} 1-(1-\alpha)(\frac{\theta_0}{\theta})^n, & \text{if } \theta > \theta_0 \\ \alpha(\frac{\theta_0}{\theta})^n, & \text{if } \theta < \theta_0 \end{cases}$

$= \begin{cases} E_\theta \phi_1(X) \\ E_\theta \phi_2(X) \end{cases}$

$\ge E_\theta \phi^*(X)$,

$\forall \phi^*$ is a level α test for H

Hence, the test $\phi(X)$ is a UMP level α test for testing $H_0: \theta = \theta_0$ vs. $H_1: \theta \ne \theta_0$

10 7.

B \ A	Yes	No	
Yes	10	30	40
No	40	20	60
	50	50	100

Let A: treatment, B = contract

To test $H_0: A \perp B$ vs. $H_1: A \not\perp B$

$E = \begin{bmatrix} E_① & E_② \\ E_③ & E_④ \end{bmatrix} = \begin{bmatrix} \frac{40\times50}{100} & \frac{40\times50}{100} \\ \frac{60\times50}{100} & \frac{60\times50}{100} \end{bmatrix}$

$= \begin{bmatrix} 20 & 20 \\ 30 & 30 \end{bmatrix}$

$\chi^2 = \frac{(10-20)^2}{20} + \frac{(30-20)^2}{20} + \frac{(40-30)^2}{30} + \frac{(20-30)^2}{30}$

$= 5+5+\frac{10}{3}+\frac{10}{3} = 16.\bar{6}$ 10

$\chi^2 = 16.\bar{6} > \chi^2_{(2-1)(2-1), 0.05} = \chi^2_{1, 0.05} = 3.84$

$\Rightarrow$ rej H_0 i.e. ...

8. To test $H_0: X \sim Gamma(\theta, 1)$, $1 \le \theta \le 6$ 0

vs. $H_1: X \sim P(\theta)$, $1 \le \theta \le 6$

No mind.