Calculus AJ
Instructor: Yu-Ling Tseng Midterm 20161123

$\star$ Answer and mark clearly the questions in the provided answer sheets.
Write down your name and student's ID on the each answer sheet you used.
$*$ **Note:** No points will be given if no arguments are provided for an answer.
For your information:

- $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x)$
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
- $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ and $\frac{d}{dx}x^r = rx^{r-1}$, for all r
- $\frac{d}{dx}\sin(x) = \cos(x)$ and $\frac{d}{dx}\cos(x) = -\sin(x)$

Good Luck! $\sim\sim$ *Yuling* ◡̈

1. (10 points) Find the constants a and b such that the function $f(x)$ is continuous on the entire real number line, where

$$f(x) = \begin{cases} 2 & x \leq -1 \\ ax + b & -1 < x < 3 \\ -2 & x \geq 3 \end{cases}$$

2. (10 points) Find the point(s), if any, at which the graph of

$$f(x) = \frac{x^2 + 8}{x - 1}$$

has a horizontal tangent line.

3. (20 points) Find the indicated limit or show it does not exist. If the limiting value is infinite, indicate whether it is ∞ or $-\infty$.

$$(a) \quad \lim_{x \to 1} \frac{\ln x}{x^2 - 1} \qquad\qquad (b) \quad \lim_{x \to \infty} x^5 e^{-3x}$$

4. (10 points) Find the equation of the tangent line to the curve of $x + y - 1 = \ln(x^2 + y^2)$ at the point $(1, 0)$.

5. (10 points) Find the absolute maximum and absolute minimum (if any) of
$f(t) = 3t^5 - 5t^3$ on the closed interval $-2 \leq t \leq 0$.

6. (40 points) Find the derivative $\frac{dy}{dx}$ or $f'(x)$ where

(a) $y\, e^{x - x^3} = 3x + y^2 \ln((x^2+1)^4)$ $\qquad$ (b) $f(x) = x^x\, 5^{x^2}$

(c) $y = \frac{(3x^2 + e^{4x})^3 e^{-4x}}{(1 + \cos(x^3) + x^2)^{2/3}}$ $\qquad$ (d) $f(x) = \frac{e^{-x^2} + x}{\log_{10} x}$

7. (10 points) Show that if a function is differentiable, then it is continuous.