

Table of Selected Discrete and Continuous Distributions and Some of Their Characteristics

Distribution	Probability density function	Mean	Variance
<b>Binomial, <math>B(n, p)</math></b>	$f(x) = \binom{n}{x} p^x q^{n-x}, x=0, 1, \dots, n;$ $0 < p < 1, q = 1 - p$	$np$	$npq$
<b>(Bernoulli, <math>B(1, p)</math>)</b>	$f(x) = p^x q^{1-x}, x=0, 1$	$p$	$pq$
<b>Poisson <math>P(\lambda)</math></b>	$f(x) = e^{-\lambda} \frac{\lambda^x}{x!}, x=0, 1, \dots; \lambda > 0$	$\lambda$	$\lambda$
<b>Hypergeometric</b>	$f(x) = \frac{\binom{m}{x} \binom{n}{r-x}}{\binom{m+n}{r}}$ , where $x=0, 1, \dots, \min(r, m)$	$\frac{mr}{m+n}$	$\frac{mr(m+n-r)}{(m+n)^2(m+n-1)}$
<b>Negative Binomial</b>	$f(x) = p \binom{r+x-1}{x} q^x, x=0, 1, \dots;$ $0 < p < 1, q = 1 - p$	$\frac{rq}{p}$	$\frac{rq}{p^2}$
<b>(Geometric</b>	$f(x) = pq^x, x=0, 1, \dots$	$\frac{q}{p}$	$\frac{q}{p^2}$
<b>Multinomial</b>	$f(x_1, \dots, x_k) = \frac{n!}{x_1! x_2! \dots x_k!} \times$ $p_1^{x_1} p_2^{x_2} \dots p_k^{x_k}, x_i \geq 0$ integers, $x_1 + \dots + x_k = n; p_j > 0, j = 1,$ $2, \dots, k, p_1 + p_2 + \dots + p_k = 1$	vector of expectations: $(np_1, \dots, np_k)'$	vector of variances: $(np_1 q_1, \dots, np_k q_k)'$ $q_j = 1 - p_j, j = 1, \dots, k$
<b>Normal, <math>N(\mu, \sigma^2)</math></b>	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right],$ $x \in \mathbb{R}; \mu \in \mathbb{R}, \sigma > 0$	$\mu$	$\sigma^2$
<b>(Standard Normal, <math>N(0, 1)</math>)</b>	$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right], x \in \mathbb{R}$	0	1)
<b>Gamma</b>	$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right), x > 0;$ $\alpha, \beta > 0$	$\alpha\beta$	$\alpha\beta^2$
<b>Chi-square</b>	$f(x) = \frac{1}{\Gamma\left(\frac{r}{2}\right) 2^{r/2}} x^{\frac{r}{2}-1} \exp\left(-\frac{x}{2}\right), x > 0;$ $r > 0$ integer	$r$	$2r$

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<b>Negative Exponential</b>	$f(x) = \lambda \exp(-\lambda x), x > 0; \lambda > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
<b>Uniform, <math>U(\alpha, \beta)</math></b>	$f(x) = \frac{1}{\beta - \alpha}, \alpha \leq x \leq \beta;$ $-\infty < \alpha < \beta < \infty$	$\frac{\alpha + \beta}{2}$	$\frac{(\alpha - \beta)^2}{12}$
<b>Beta</b>	$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}, 0 < x < 1;$ $\alpha, \beta > 0$	$\frac{\alpha}{\alpha + \beta}$	$\frac{\alpha\beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)}$
<b>Cauchy</b>	$f(x) = \frac{\sigma}{\pi} \frac{1}{\sigma^2 + (x - \mu)^2}, x \in \mathbb{R};$ $\mu \in \mathbb{R}, \sigma > 0$	Does not exist	Does not exist
<b>Bivariate Normal</b>	$f(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left(-\frac{q}{2}\right),$ $q = \frac{1}{1-\rho^2} \left[ \left(\frac{x_1 - \mu_1}{\sigma_1}\right)^2 - 2\rho \left(\frac{x_1 - \mu_1}{\sigma_1}\right) \right.$ $\left. \times \left(\frac{x_2 - \mu_2}{\sigma_2}\right) + \left(\frac{x_2 - \mu_2}{\sigma_2}\right)^2 \right],$ $x_1, x_2, \in \mathbb{R};$ $\mu_1, \mu_2 \in \mathbb{R}, \sigma_1, \sigma_2 > 0, -1 \leq \rho \leq 1$	vector of expectations: $(\mu_1, \mu_2)'$	vector of variances: $(\sigma_1^2, \sigma_2^2)'$
<b>k-Variate Normal, <math>N(\mu, \Sigma)</math></b>	$f(\mathbf{x}) = (2\pi)^{-k/2}  \Sigma ^{-1/2} \times$ $\exp\left[-\frac{1}{2}(\mathbf{x} - \mu)\Sigma^{-1}(\mathbf{x} - \mu)\right],$ $\mathbf{x} \in \mathbb{R}^k; \mu \in \mathbb{R}, \Sigma: k \times k$ non-singular symmetric matrix	mean vector: $\mu$	covariance matrix: $\Sigma$

$$E(\alpha+1) = \alpha E(\alpha), \forall \alpha > 0$$

$$E(1) = 1.$$

$$E(n+1) = n! \quad \forall n = \text{positive integer}$$