

1. Evaluate the improper integrals

$$(a) \int_{-8}^1 \frac{1}{x^{1/3}} dx \quad (b) \int_{-\infty}^0 x e^x dx \quad (c) \int_0^1 x \ln x dx$$

$$(d) \int_{-\infty}^{\infty} 2x e^{-x^2} dx \quad (e) \int_{-1}^4 \frac{1}{\sqrt{|x|}} dx \quad (f) \int_0^2 \frac{1}{\sqrt{|x-1|}} dx$$

2. Use integration, the Direct Comparison Test, or the Limit Comparison Test to test the integrals for convergence

$$(a) \int_0^1 \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx \quad (b) \int_1^{\infty} \frac{\sqrt{x+1}}{x^2} dx \quad (c) \int_{\pi}^{\infty} \frac{2 + \cos x}{x} dx$$

$$(d) \int_{\pi}^{\infty} \frac{1 + \sin x}{x^2} dx \quad (e) \int_1^{\infty} \frac{e^x}{x} dx \quad (f) \int_{-\infty}^{\infty} \frac{1}{\sqrt{x^4 + 1}} dx$$

3. An example for  $\int_{-\infty}^{\infty} f(x) dx$  may not equal to  $\lim_{b \rightarrow \infty} \int_{-b}^b f(x) dx$ .

Show that

$$\int_0^{\infty} \frac{2x}{x^2 + 1} dx$$

diverges and hence that

$$\int_{-\infty}^{\infty} \frac{2x}{x^2 + 1} dx$$

diverges. Then show that

$$\lim_{b \rightarrow \infty} \int_{-b}^b \frac{2x}{x^2 + 1} dx = 0.$$