⋆ Answer and mark clearly the questions in the provided answer sheets.
Write down your name and student's ID on the each answer sheet you used.
∗ **Note:** There are 115 points in this exam.
No points will be given if no arguments are provided for an answer.
Good Luck! ∼∼ *Yuling* ⌣̈

1. (10 points) Let $X_1, \ldots, X_n$ be an *i.i.d* sample from $P(\lambda)$, $\lambda > 0$. Show that the Fisher information number $(X_1, \ldots, X_n)$ contains about $\sqrt{\lambda}$ is independent of λ.

2. (10 points) Let $X_1, \ldots, X_n$ be *i.i.d* random variables from the Gamma(α, β), find a MOME (method of moments estimator) of α/β.

3. (10 points) Let $X_1, \ldots, X_n$ be *i.i.d* random variables with p.d.f. $f(\cdot; \theta_1, \theta_2)$ given by
$$f(x; \theta_1, \theta_2) = \frac{1}{\theta_2} \exp\left(-\frac{x-\theta_1}{\theta_2}\right), \quad x \geq \theta_1, \quad \theta = (\theta_1, \theta_2)' \in \Omega = R \times (0, \infty).$$
Find the MLE (maximum likelihood estimator) for $\eta(\theta) = \theta_1 \times \sqrt{\theta_2}$.

4. (10 points) Let X be a random variable with one of these *p.d.f.'s*:
if $\theta = 0$ then
$$f(x;\theta) = \begin{cases} 2 - 4x & \text{if } 0 < x < 1/2, \\ 4x - 2 & \text{if } 1/2 \leq x < 1; \end{cases}$$
if $\theta = 1$ then
$$f(x;\theta) = \begin{cases} 2(1-x) & \text{if } 0 < x < 1, \\ 0 & \text{otherwise}; \end{cases}$$
while if $\theta = 2$
$$f(x;\theta) = \begin{cases} 1 & \text{if } 0 < x < 1, \\ 0 & \text{otherwise}. \end{cases}$$
Find the MLE (maximum likelihood estimator) of $\eta(\theta) = \theta^2$ and check if it is unbiased.

5. (10 points) Let $X_1, \ldots, X_n$, $n > 3$, be an *i.i.d* sample from $B(1, p)$, $p \in [0, 1]$. Find the UMVUE (uniformly minimum variance unbiased estimator) of $p(1-p)^2$.

6. $X_1, \ldots, X_n$ be an *i.i.d* sample from $N(\mu, \sigma^2)$, both $\mu \in R$ and $\sigma > 0$ are unknown. Let $\bar{X} = \sum_{i=1}^n X_i/n$ and $S^2 = \sum_{i=1}^n (X_i - \bar{X})^2/(n-1)$.
   (a) (10 points) Show that the statistic $T(X) = (\bar{X}, S^2)$ is sufficient for (μ, σ^2) and is complete.
   (b) (10 points) Find the UMVUE for μ^2.
   (c) (10 points) Now, if $\sigma^2 = \mu^2$, hence the only unknown parameter is μ. Is the statistic $T(X)$ defined in (a) still sufficient? Is it complete?

7. (15 points) Let $X_1, \ldots, X_n$, $n > 1$, be *i.i.d.* random variables with *p.d.f.*
$f(x;\theta) = \theta \exp(-\theta\, x)$, $x > 0$, $\theta \in \Omega = (0, \infty)$.
Find the UMVUE of θ.
Investigate if the Cramér-Rao bound is attained.

8. (20 points) Let $X_1, \ldots, X_n$ be *i.i.d* random variables from $U[0, \theta]$, $\theta > 0$.
Find the UMVUE, MLE for θ, respectively; compare them by calculating their MSE's (mean squared error).