Handout From Hicks.

Design, Tsang

Graphical Representation of the Gun-Loading Experiment

The gun-loading experiment discussed in Example 7.1 is represented graphically in Figure 7.6, which depicts the difference between crossed and nested factors. The lines extending from the levels of method to those of group cross, indicating that each level of method can be used in combination with any level of group. On the other hand, the lines originating at a particular group extend directly to the teams within that group and do not cross over to teams within another group. This is because the teams within a group are unique to that group, whereas the grouping levels can occur in both methods.

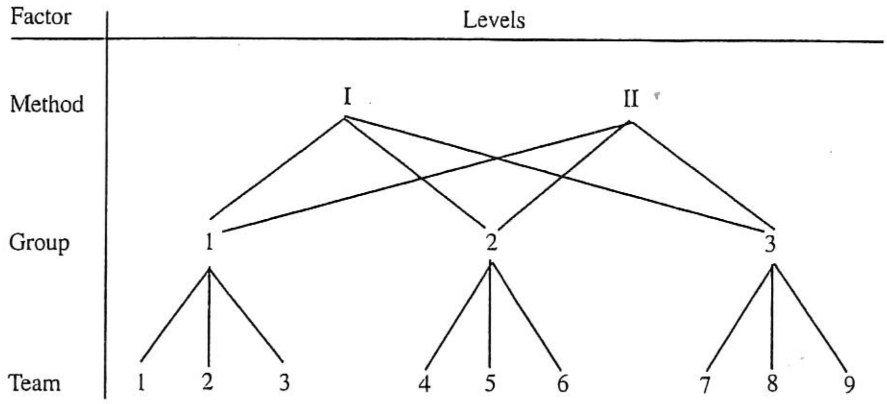

Figure 7.6 The crossed and nested factors of Example 7.1.

7.4 NESTED-FACTORIAL EXPERIMENTS

Many multiple-factor experiments involve both factors that are crossed with others and factors that are nested within levels of the others. When this happens—that is, when both factorial and nested factors appear in the same experiment—the term *nested-factorial experiment* is applied. The analysis of such an experiment is simply an extension of the methods of Chapter 5 and this chapter. Care must be exercised, however, in computing some of the interactions. Levels of both factorial and nested factors may be either fixed or random. The methods of Chapter 6 can be used to determine the EMS values and the proper tests to be run.

Example 7.1

An investigator who wished to increase the number of rounds per minute that could be fired from a naval gun devised a new loading method (method I) with the intent of improving performance in this task over that obtained with the existing method of loading (method II). Realizing that the general physique of a person might affect the speed with which a gun could be loaded, the investigator selected artillery teams of persons in three general groupings (slight, average, and heavy). The classification of such persons was on the basis of an Armed Services classification table. Three teams were randomly chosen to represent each of the three physique groupings.

Once these three teams were selected to represent each of the physique groups, the nine teams set up a schedule for coming to a gym to run the test. In each case, a coin was tossed when a team arrived at the gym to decide which of the two methods would be used that day. This was repeated for three more random days with the restriction that in the four days each team should be assigned to each method twice.

To collect the data, a team would be timed each day for 20 minutes. Due to start-up concerns and fatigue concerns, only the middle 10 minutes would be used to give the number of rounds per minute for that run. The model for this experiment was:

$$Y_{ijkm} = \mu + M_i + G_j + MG_{ij} + T_{k(j)} + MT_{ik(j)} + \varepsilon_{m(ijk)}$$

where

$$\begin{aligned} M_i &= \text{methods, } i = 1, 2 \\ G_j &= \text{groups, } j = 1, 2, 3 \\ T_{k(j)} &= \text{teams within groups, } k = 1, 2, 3, \text{ for all } j \\ \varepsilon_{m(ijk)} &= \text{random error, } m = 1, 2, \text{ for all } i, j, k \end{aligned}$$

The EMS values are shown in Table 7.7, which indicates the proper F tests to run, and the data appear in Table 7.8.

An ANOVA summary for this experiment is displayed in Table 7.9. Notice that the expected mean squares summarized are identical to those in Table 7.7. The results show a very significant method effect (p value = 0.000): as indicated in Table 7.8, the new method (I) averaged 23.58 rounds per minute and the old method (II) averaged only 15.08 rounds per minute. The results also show a significant difference among teams within groups (p value = 0.040), which points out individual differences in personnel. No other effects or interactions are significant.

TABLE 7.7
EMS for Gun-Loading Problem

< exercise 1 >

Source	2 F *i*	3 F *j*	3 R *k*	2 R *m*	EMS
M_i	0	3	3	2	$\sigma_\varepsilon^2 + 2\sigma_{MT}^2 + 18\phi_M$
G_j	2	0	3	2	$\sigma_\varepsilon^2 + 4\sigma_T^2 + 12\phi_G$
MG_{ij}	0	0	3	2	$\sigma_\varepsilon^2 + 2\sigma_{MT}^2 + 6\phi_{MG}$
$T_{k(j)}$	2	1	1	2	$\sigma_\varepsilon^2 + 4\sigma_T^2$
$MT_{ik(j)}$	0	1	1	2	$\sigma_\varepsilon^2 + 2\sigma_{MT}^2$
$\varepsilon_{m(ijk)}$	1	1	1	1	σ_ε^2

TABLE 7.8
Data and ANOVA for Gun-Loading Problem

	Group								
	I			II			III		
Team	1	2	3	4	5	6	7	8	9
Method I	20.2	26.2	23.8	22.0	22.6	22.9	23.1	22.9	21.8
	24.1	26.9	24.9	23.5	24.6	25.0	22.9	23.7	23.5
Method II	14.2	18.0	12.5	14.1	14.0	13.7	14.1	12.2	12.7
	16.2	19.1	15.4	16.1	18.1	16.0	16.1	13.8	15.1

Manual Calculation of Sums of Squares

TABLE 7.10
Data on Gun-Loading Problem for Group I

	Team 1		Team 2		Team 3		Method Totals
Method I	20.2 24.1	44.3	26.2 26.9	53.1	23.8 24.9	48.7	√ 146.1
Method II	14.2 16.2	30.4	18.0 19.1	37.1	12.5 15.4	27.9	95.4
Team totals		74.7		90.2		76.6	241.5

TABLE 7.11
Data on Gun-Loading Problem for Group II

	Team 4		Team 5		Team 6		Method Totals
Method I	22.0 23.5	45.5	22.6 24.6	47.2	22.9 25.0	47.9	√ 140.6
Method II	14.1 16.1	30.2	14.0 18.1	32.1	13.7 16.0	29.7	92.0
Team totals		75.7		79.3		77.6	232.6

SS_M

√ 424.6

271.4

TABLE 7.12
Data on Gun-Loading Problem for Group III

	Team 7		Team 8		Team 9		Method Totals
Method I	23.1 22.9	46.0	22.9 23.7	46.6	21.8 23.5	45.3	√ 137.9
Method II	14.1 16.1	30.2	12.2 13.8	26.0	12.7 15.1	27.8	84.0
Team totals		76.2		72.6		73.1	221.9

696.

$$SS_M = \frac{424.6^2 + 271.4^2}{18} - \frac{696^2}{36} = 651.951$$

$$SS_G = \frac{241.5^2 + 232.6^2 + 221.9^2}{12} - \frac{696^2}{36} = 16{,}052$$