Outline

6.3 Paired-Category Ordinal Logits

6.3.2 Example: Political Ideology Revisited

The party a liation e ect :

- $\hat{\beta} = 0.435$
- Odds is $\exp(\hat{\beta}) = 1.54$

VL v.s. VC, $b = 1, a = 5$

$\Rightarrow$ Odds is exp[0.435(5-1)] = $(1.54)^4$ = 5.7

- The probability of a dead fetus.
  - $\log(\frac{p_1}{p_2+p_3})$

Concentration	Dead	Malformation	Normal
0	15	1	281
62.5	17	0	225
125	22	7	283
250	38	59	202
500	144	132	9

- The conditional probability of a malformed fetus, given that the fetus was alive.
  - $\log(\frac{p_2}{p_3})$

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0		1	281
62.5		0	225
125		7	283
250		59	202
500		132	9

Simultaneous tting model

$$\log\left(\frac{p_1}{p_2+p_3}\right) = a_1 + b_1 x \quad \text{and} \quad \log\left(\frac{p_2}{p_3}\right) = a_2 + b_2 x$$

- $G^2 = 11.8$, df=6 $\quad G^2 = G_1^2 + G_2^2$
- $\hat{b}_1 = 0.0064$
- $\hat{b}_2 = 0.0174$

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Note: Same "results".

6.3.5 Overdispersion in Clustered Data

Statistical dependence

Heterogeneous probabilities

,! Violates the binomial assumption

6.4 Tests of Conditional Independence

Test the hypothesis of conditional independence

Ordinal v.s. Ordinal : Cumulative logit model

Nominal v.s. Nominal : Baseline-category logit model

Ordinal v.s. Ordinal

The test statistic generalizes the correlation statistic for two-way tables.

Generalized correlation statistic $\rightarrow \chi_1^2$

Ref.: Mantel. 1963. Chi-square tests with one degree of freedom: Extensions of the Mantel-Haenszel procedure.

Di erent choices of Score

Job satisfaction : (1, 3, 4, 5)
Income : (3, 10, 20, 35)

Job satisfaction : (1, 2, 3, 4)
Income : (1, 2, 3, 4)

Q: Get the same conclusion?

Ordinal v.s. Ordinal

Income : (3, 10, 20, 35), Job satisfaction : (1, 3, 4, 5)

- $r_F = 0.16$, $r_M = 0.37$
- Generalized correlation statistic = 6.2, df = 1

  $\Rightarrow$ P-value =0.01(i.e. $X \not\perp Y | Z$)

  Note: same conclusion as previous subsection.

Ordinal v.s. Ordinal

Income : (3, 10, 20, 35), Job satisfaction : (1, 3, 4, 5)

- r

Nominal v.s. Ordinal

Job satisfaction : (1, 2, 3, 4)

The mean job satisfaction at the four levels of income

F : (2.82, 2.84, 3.29, 3.00) M : (2.60, 2.78, 3.30, 3.31)

e.g. 17 females with income < 5000 :

$$[1(1) + 2(3) + 3(11) + 4(2)]/17 = 2.82$$

		Job Satisfaction			
Gender	Income	VD	ALS	MS	VS
Female	< 5000	1	3	11	2

Nominal v.s. Ordinal

Income : Nominal

Job satisfaction : (1, 2, 3, 4)

Generalized CMH statistic ! $c^2_{I \quad 1}$

Test whether the row mean scores di er

- Generalized CMH statistic = 9.2, df = 3

  =) P-value = 0.03

Ref.: Landis. 1978. Average partial association in three-way contingency table: A review and discussion of alternative tests.

Nominal v.s. Nominal

General association test ! $c^2_{(I \quad 1)(J \quad 1)}$

Detect *any* type of association

- Generalized association statistic = 10.2, df = 9

  =)(P-value)=0.34

Ref.: Landis. 1998. Mantel-Haenszel methods.
Stokes. 2000. *Categorical Data Analysis Using the SAS System*, 2nd ed.