榮譽第一

國立東華大學 應用數學系 ______學年度第____學期 考試科目：______ □期中□期末考試試卷

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1. $\lim_{x\to c} f(x)-f(c) = \lim_{x\to 0} \frac{f(x)-f(c)}{x-c}\cdot(x-c)$

$= \lim_{x\to c} \frac{f(x)-f(c)}{x-c}\cdot \lim_{x\to c}(x-c)$

$\because \lim_{x\to c} \frac{f(x)-f(c)}{x-c} = f'(c)$ exist.

$\lim_{x\to c}(x-c) = 0$ exist.

$\therefore = f'(c)\cdot 0 = 0$

$(\Leftarrow)\ \lim_{x\to c} f(x) = f(c)$

i.e. f is conti. at c.

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2. $f'(x) = -x^2+2x-1 = -(x^2-2x+1) = -(x-1)^2$

let $f'(x)=0$, $x=1$

$-\ |\ -$ $\Rightarrow$ no extrema.

$f''(x) = -2(x-1)\cdot 1 = -2(x-1)$

$+\ |\ -$ $\Rightarrow f(1)$ is inflection point

$\therefore$ (a) $f(x)$ is decreasing #

(b.) $f(x)$ is c.u on $(-\infty,1)$ #

is c.D on $(1,\infty)$ #

(c.) no extrema ; inflection points: $(1,f(1))$ #

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3. $(2x+xy-2)' = (\ln(x^3+y^2))'$, $(1,0)$

$2+y+\frac{dy}{dx}\cdot x = \frac{3x^2+2y\cdot\frac{dy}{dx}}{x^3+y^2}$

$\frac{dy}{dx}\left(x-\frac{2y}{x^3+y^2}\right) = \frac{3x^2}{x^3+y^2}-y-2$

$\xrightarrow[\text{代入}]{(1,0)} \frac{dy}{dx} = \left(\frac{3}{1+0}-0-2\right)\div\left(1-\frac{0}{1+0}\right) = 1\div 1 = 1$

$\therefore y-0 = 1(x-1)$, $y=x-1$ #

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4. $h(t) = (e^{-t}+e^{t})^3$, $-2\le t\le 3$

$h'(t) = 3(e^{-t}+e^{t})^2\cdot(-e^{-t}+e^{t}) \overset{\text{let}}{=} 0$, $e^{-t}=e^{t}=e^{0}$

$t=0$

$h(0) = (e^0+e^0)^3 \to$ abs. min #

$h(-2) = (e^2+e^{-2})^3$

$h(3) = (e^{-3}+e^3)^3 \to$ abs. max #

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5. (a.) $ye^{5x-x^3} = 5\sin(4x)+y^2\ln((2x^3+5)^2)+\log_5 y$

$\xrightarrow{\frac{d}{dx}} \frac{dy}{dx}\cdot e^{5x-x^3}+e^{5x-x^3}\cdot(5-3x^2)\cdot y = 5\cos(4x)\cdot 4$

$+2y\frac{dy}{dx}\cdot 2\cdot\ln(2x^3+5)+2y^2\cdot\frac{6x^2}{2x^3+5}+\frac{1}{y\log 5}\cdot\frac{dy}{dx}$

$\frac{dy}{dx}\Big[\underbrace{e^{5x-x^3}-4y\ln(2x^3+5)-\frac{1}{y\log 5}}_{\text{set } B}\Big] = \Big[\underbrace{20\cos(4x)+\frac{12x^2y^2}{2x^3+5}-y(5-3x^2)\cdot e^{5x-x^3}}_{\text{set } A}\Big]$

$\frac{dy}{dx} = \frac{A}{B}$ #

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(b.) $f(x) = x^x 6^{x^3} = y$

$\ln y = x\ln x + x^3\ln 6$

$\xrightarrow{\frac{d}{dx}} \frac{1}{y}\cdot\frac{dy}{dx} = \ln x+\frac{1}{x}\cdot x+3x^2\ln 6+0$

$\frac{dy}{dx} = y(\ln x+1+3x^2\ln 6)$

$= x^x 6^{x^3}(\ln x+1+3x^2\ln 6)$ #

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(c.) $f(x) = 5\sec^2(\ln(3\sqrt{x}))$

$f'(x) = 5\cdot 2\sec(\ln(3\sqrt{x}))\cdot\tan(\ln(3\sqrt{x}))$

$\cdot\sec(\ln(3\sqrt{x}))\cdot\frac{1}{3\sqrt{x}}\cdot\frac{3}{2}\cdot\frac{1}{\sqrt{x}}\cdot 1$

$f'(x) = 10\sec(\ln(3\sqrt{x}))\cdot\tan(\ln 3\sqrt{x})\cdot$

$\sec(\ln(3\sqrt{x}))\cdot\frac{1}{2x}$ #

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(d.) $f(x) = \frac{(5x-1)(7x-2)(8x-3)(3x-4)}{(5x+1)(7x+2)(8x+3)(3x+4)} = y$

$\xrightarrow{\text{取}\ln} \ln y = \ln(5x-1)+\ln(7x-2)+\ln(8x-3)+\ln(3x-4)$

$-\ln(5x+1)-\ln(7x+2)-\ln(8x+3)-\ln(3x+4)$

$\xrightarrow{\frac{d}{dx}} \frac{1}{y}\cdot\frac{dy}{dx} = \frac{5}{5x-1}+\frac{7}{7x-2}+\frac{8}{8x-3}+\frac{3}{3x-4}-\frac{5}{5x+1}$

$-\frac{7}{7x+2}-\frac{8}{8x+3}-\frac{3}{3x+4}$

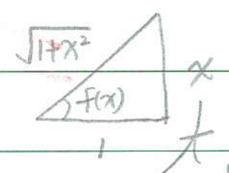
$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{25x+5-25x+5}{(5x-1)(5x+1)}+\frac{49x+14-49x+14}{(7x-2)(7x+2)}+\frac{64x+24-64x+24}{(8x+3)(8x-3)}+\frac{9x+12-9x+12}{(3x-4)(3x+4)}$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{10}{25x^2-1}+\frac{28}{49x^2-4}+\frac{48}{64x^2-9}+\frac{24}{9x^2-16}$

$\frac{dy}{dx} = \left(\frac{10}{25x^2-1}+\frac{28}{49x^2-4}+\frac{48}{64x^2-9}+\frac{24}{9x^2-16}\right)\cdot y$ #

8 (e.) $f(x)=\tan^{-1}(x) \Leftrightarrow \tan(f(x))=x$

$\xrightarrow{\frac{d}{dx}}$ $\sec^2(f(x))\cdot f'(x)=1$



$f'(x)=\dfrac{1}{\sec^2(f(x))}$

$=\dfrac{1}{(\sqrt{1+x^2})^2}$ $\qquad \sec(x)=\dfrac{1}{\cos(x)}=\dfrac{1}{\frac{1}{\sqrt{1+x^2}}}=\sqrt{1+x^2}$

$=\dfrac{1}{1+x^2}$ #

8 6. (a). $\lim\limits_{x\to 0}(e^{-2x}-3x)^{\frac{2}{x}}$

$\exp\left(\lim\limits_{x\to 0}\frac{2}{x}\ln(e^{-2x}-3x)\right)=\exp(-10)=e^{-10}$ #

$=\lim\limits_{x\to 0}\dfrac{2\ln(e^{-2x}-3x)}{x}=\lim\limits_{x\to 0}\dfrac{2\times\left(\frac{-2e^{-2x}-3}{e^{-2x}-3x}\right)}{1}$

$=2\times(-5)=-10$

8 (b) $\lim\limits_{x\to 0}\dfrac{\sin(7x)}{\sin(\sqrt{\pi}x)}$

$=\lim\limits_{x\to 0}\dfrac{7\cos(7x)}{\cos(\sqrt{\pi}x)\cdot\sqrt{\pi}}$

$=\dfrac{7}{\sqrt{\pi}}$ #

8 (c) $\lim\limits_{t\to\infty}t^5 e^{-2t}$

$=\lim\limits_{t\to\infty}\dfrac{t^5}{e^{2t}}$

$=\lim\limits_{t\to\infty}\dfrac{5t^4}{2e^{2t}}=\lim\limits_{t\to\infty}\dfrac{20t^3}{4e^{2t}}=\lim\limits_{t\to\infty}\dfrac{60t^2}{8e^{2t}}$

$=\lim\limits_{t\to\infty}\dfrac{120t}{16e^{2t}}=\lim\limits_{t\to\infty}\dfrac{120}{32e^{2t}}=0$ #

8 7. $f(x)=|x|^x=\begin{cases}x^x, & x>0\\ (-x)^x, & x<0\end{cases}$) x在0不連續

① $\lim\limits_{x\to 0^+}|x|^x=\lim\limits_{x\to 0^+}x^x=\exp\left(\lim\limits_{x\to 0^+}x\ln x\right)=\exp(0)=e^0$

$\lim\limits_{x\to 0^+}x\ln x=\lim\limits_{x\to 0^+}\dfrac{\ln x}{\frac{1}{x}}=\lim\limits_{x\to 0^+}\dfrac{\frac{1}{x}\cdot 1}{-\frac{1}{x^2}}=\lim\limits_{x\to 0^+}\dfrac{-x^2}{x}=0$

② $\lim\limits_{x\to 0^-}|x|^x=\lim\limits_{x\to 0^-}(-x)^x=\exp\left(\lim\limits_{x\to 0^-}x\ln(-x)\right)=\exp(0)=e^0$

$\lim\limits_{x\to 0^-}x\ln(-x)=\lim\limits_{x\to 0^-}\dfrac{\ln(-x)}{\frac{1}{x}}=\lim\limits_{x\to 0^-}\dfrac{-\frac{1}{x}\cdot(-1)}{-\frac{1}{x^2}}=\lim\limits_{x\to 0^-}\frac{1}{x}\cdot(-x^2)=0$

從①.②知 $\lim\limits_{x\to 0^+}f(x)=\lim\limits_{x\to 0^-}f(x)=e^0=1$

limit exist

$\Rightarrow$ f(x) is conti. on $(-\infty,0)$, $(0,\infty)$, $\because$ x^x, $x>0$, $(-x)^x$, $x<0$ are diff.

and f(0) is a removable point # hence conti.

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1. 已知 $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}=f'(c)$ 存在

$\because \lim_{x\to c} f(x)=\lim_{x\to c}\left[\frac{f(x)-f(c)}{(x-c)}\cdot(x-c)+f(c)\right]$

又 $\frac{f(x)-f(c)}{x-c}$ 存在, $\lim_{x\to c}(x-c)=0$ 存在 且 $\lim_{x\to c} f(c)=f(c)$ 存在

$\therefore \lim_{x\to c} f(x)=\lim_{x\to c}\left[\frac{f(x)-f(c)}{(x-c)}\cdot(x-c)\right]+\lim_{x\to c} f(c)$

$=f'(c)\cdot 0+f(c)$

$=f(c)$, 故得證 #

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2. $f'(x)=-x^2+2x-1=-(x^2-2x+1)=-(x-1)^2$

(a) $f(1)=\frac{14}{3}$

$f(x)$	↘	↘
$f'(x)$	$-$	$-$

$x=1$

$\therefore f(x)$ is decreasing on $(-\infty,1),(1,\infty)$

(b) $f''(x)=-2x+2=-2(x-1)$

$f(1)=\frac{14}{3}$

$f(x)$	$\cup$	$\cap$
$f''(x)$	$+$	$-$

$x=1$

$\therefore f$ is concave upward on $(-\infty,1)$

f is concave downward on $(1,\infty)$ #

(c) 根據 (a), f has no relative extrema

根據 (b), inflection points : $(1,\frac{14}{3})$ #

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3. $\frac{d}{dx}(2x+xy-2)=\frac{d}{dx}(\ln(x^3+y^2))$

$\Rightarrow 2+x\left(\frac{dy}{dx}\right)+y=\frac{1}{x^3+y^2}\times\left(3x^2+2y\left(\frac{dy}{dx}\right)\right)$

$\left(x-\frac{2y}{x^3+y^2}\right)\left(\frac{dy}{dx}\right)=\frac{3x^2}{x^3+y^2}-2-y$

$$\frac{dy}{dx}=\frac{\frac{3x^2}{x^3+y^2}-2-y}{x-\frac{2y}{x^3+y^2}}$$

$\Rightarrow (1,0)$ 代入 $\frac{dy}{dx}$: $\frac{3-2-0}{1-0}=1=$ slope at $(1,0)$

$\therefore$ tangent line : $y=x-1$ #

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4. $h'(t)=3(e^{-t}+e^{t})^2(-e^{-t}+e^{t})=0$, $(e>2)$ $\Rightarrow t=0$

$h(-2)=(e^2+e^{-2})^3>8$　$h(0)=8$　$h(3)=(e^{-3}+e^3)^3>(e^2+e^{-2})^3>8$

$h(t)$	↘	↗
$h'(t)$	$-$	$+$

$t=-2$　$\boxed{t=0}$　$t=3$

$\therefore$ absolute maximum : $h(3)=(e^{-3}+e^3)^3$

absolute minimum : $h(0)=8$ #

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5. (a) $\Rightarrow ye^{5x-x^3}=5\sin(4x)+y^2\cdot 2\ln(2x^3+5)+\frac{\ln y}{\ln 5}$

$\xrightarrow{\frac{d}{dx}}$ $e^{5x-x^3}\left(\frac{dy}{dx}\right)+(5-3x^2)ye^{5x-x^3}=5\cos(4x)\cdot 4$

$+2y\cdot 2\ln(2x^3+5)\left(\frac{dy}{dx}\right)$

$+y^2\cdot 2\cdot\frac{6x^2}{2x^3+5}$

$+\frac{\frac{1}{y}\left(\frac{dy}{dx}\right)\ln 5}{(\ln 5)^2}$

$\Rightarrow \left(e^{5x-x^3}-4y\ln(2x^3+5)-\frac{1}{y\ln 5}\right)\left(\frac{dy}{dx}\right)=20\cos(4x)+\frac{12x^2y^2}{2x^3+5}$

$-(5-3x^2)ye^{5x-x^3}$

$$\Rightarrow \left(\frac{dy}{dx}\right)=\frac{20\cos(4x)+\frac{12x^2y^2}{2x^3+5}-(5-3x^2)ye^{5x-x^3}}{e^{5x-x^3}-4y\ln(2x^3+5)-\frac{1}{y\ln 5}}$$ #

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5. (b) $\Rightarrow \ln f(x) = \ln(x^x \cdot 6^{x^3}) = x\ln x + x^3 \ln 6$

$\xrightarrow{\frac{d}{dx}} \frac{f'(x)}{f(x)} = \ln x + x \cdot \frac{1}{x} + 3x^2 \ln 6$

$f'(x) = f(x)(\ln x + 1 + 3x^2 \ln 6)$

$= (x^x 6^{x^3})(\ln x + 1 + 3x^2 \ln 6)$ #

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5. (c) $f'(x) = 10\sec(\ln(3\sqrt{x})) \cdot [\tan(\ln(3\sqrt{x}))\sec(\ln(3\sqrt{x}))] \cdot \frac{3 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}}}{3\sqrt{x}}$

$= \cancel{10}^{5}\sec(\ln(3\sqrt{x})) \cdot \tan(\ln(3\sqrt{x})) \cdot \sec(\ln(3\sqrt{x})) \cdot \frac{3}{\cancel{6}x}$

$= \left(\frac{5}{x}\right)\sec^2(\ln(3\sqrt{x}))\tan(\ln(3\sqrt{x}))$ #

5. (d) $\Rightarrow \ln f(x) = \ln\left[\frac{(5x-1)(7x-2)(8x-3)(3x-4)}{(5x+1)(7x+2)(8x+3)(3x+4)}\right]$

$= \ln(5x-1) + \ln(7x-2) + \ln(8x-3) + \ln(3x-4)$

$- \ln(5x+1) - \ln(7x+2) - \ln(8x+3) - \ln(3x+4)$

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$\xrightarrow{\frac{d}{dx}} \frac{f'(x)}{f(x)} = \frac{5}{5x-1} + \frac{7}{7x-2} + \frac{8}{8x-3} + \frac{3}{3x-4} - \frac{5}{5x+1}$

$- \frac{7}{7x+2} - \frac{8}{8x+3} - \frac{3}{3x+4}$

$= \left(\frac{5}{5x-1} - \frac{5}{5x+1}\right) + \left(\frac{7}{7x-2} - \frac{7}{7x+2}\right) + \left(\frac{8}{8x-3} - \frac{8}{8x+3}\right)$

$+ \left(\frac{3}{3x-4} - \frac{3}{3x+4}\right)$

$= \frac{10}{25x^2-1} + \frac{28}{49x^2-4} + \frac{48}{64x^2-9} + \frac{24}{9x^2-16}$

$\therefore f'(x) = f(x) \cdot \left(\frac{10}{25x^2-1} + \frac{28}{49x^2-4} + \frac{48}{64x^2-9} + \frac{24}{9x^2-16}\right)$ #

8

5. (e) $\tan(f(x)) = \tan(\tan^{-1}(x)) = x$

$\xrightarrow{\frac{d}{dx}} \sec^2(f(x)) \cdot f'(x) = 1$

$\left(\frac{1}{\cos(f(x))}\right)^2 \cdot f'(x) = 1$

$\sqrt{x^2+1}$, x, 1, $f(x)$

$f'(x) = 1 \cdot (\cos(f(x)))^2$

$= \left(\frac{1}{\sqrt{x^2+1}}\right)^2$

$= \frac{1}{x^2+1}$ #

8

6. (a) $\lim_{x\to 0}(e^{-2x} - 3x)^{\frac{2}{x}}$

$= \lim_{x\to 0} \exp(\ln(e^{-2x} - 3x)^{\frac{2}{x}})$

$= e^{\lim_{x\to 0} \ln(e^{-2x}-3x)^{\frac{2}{x}}}$

$\rightarrow \lim_{x\to 0} \ln(e^{-2x} - 3x)^{\frac{2}{x}}$

$= \lim_{x\to 0}\left(\frac{2\ln(e^{-2x}-3x) \to 0}{x \to 0}\right)$ is "$\frac{0}{0}$"

$= \lim_{x\to 0}\left[2 \cdot \left(\frac{-2e^{-2x}-3}{e^{-2x}-3x}\right)\right]$

$= 2 \times \frac{-2-3}{1-0} = -10$

$\therefore \lim_{x\to 0}(e^{-2x} - 3x)^{\frac{2}{x}} = e^{-10}$ #

8

6. (b) $\lim_{x\to 0} \frac{\sin(7x) \to 0}{\sin(\sqrt{\pi}x) \to 0}$ is "$\frac{0}{0}$"

$= \lim_{x\to 0} \frac{\cos(7x) \cdot 7}{\cos(\sqrt{\pi}x) \cdot \sqrt{\pi}}$

$= \frac{7}{\sqrt{\pi}}$ #

8

6. (c) $\lim_{t\to\infty} t^5 e^{-2t}$

$= \lim_{t\to\infty} \frac{t^5 \to \infty}{e^{2t} \to \infty}$ is "$\frac{\infty}{\infty}$"

$= \lim_{t\to\infty} \frac{5t^4 \to \infty}{2e^{2t} \to \infty}$ is "$\frac{\infty}{\infty}$"

$= \lim_{t\to\infty} \frac{20t^3 \to \infty}{4e^{2t} \to \infty}$ is "$\frac{\infty}{\infty}$"

$= \lim_{t\to\infty} \frac{60t^2 \to \infty}{8e^{2t} \to \infty}$ is "$\frac{\infty}{\infty}$"

$= \lim_{t\to\infty} \frac{120t \to \infty}{16e^{2t} \to \infty}$ is "$\frac{\infty}{\infty}$"

$= \lim_{t\to\infty} \frac{120}{32e^{2t}}$

$= 0$ #

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7. $f(x) = |x|^x$ $= \begin{cases} x^x & x>0 \\ (-x)^x & x<0 \end{cases}$

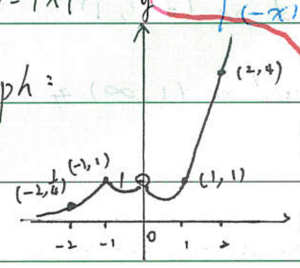
c.f. $\frac{d}{dx} x^x = \frac{d}{dx} e^{x\ln x}$

$= x^x(1+\ln x)$

$\Rightarrow f'(x)$ exists $\forall x \neq 0$

hence conti. yes

graph:



? $f(x)$ is continuous on $(0,\infty)$ and $(-\infty,0)$

There is a discontinuous ~~unremovable~~ removable at $x=0$ #

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$\lim_{x\to 0} f(x) = 1$ discussed in class.