榮譽第一

國立東華大學
應用數學系 _____學年度第___學期　考試科目：_____ □期中□期末考試試卷

學號 411132014 姓名 莊聖德 系所 企管 班別 一 任課教師：_____ 共___張

1. (a) $f'(t) = 2t\sqrt{t+2} + (t^2-9)\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{t+2}}$

$= 2t\sqrt{t+2} + \frac{t^2-9}{2\sqrt{t+2}}$

$= \frac{4t(t+2)+t^2-9}{2\sqrt{t+2}}$

$= \frac{5t^2+8t-9}{2\sqrt{t+2}}$

slope $= f'(-1) = \frac{5-8-9}{2} = -6$

$\rightarrow y = -6t - 14$ #

1. (b) $f(x) = \frac{18x^2+3x-10}{2x-3}$

$f'(x) = \frac{(36x+3)(2x-3)-(18x^2+3x-10)\cdot 2}{(2x-3)^2}$

$= \frac{72x^2-102x-9-36x^2-6x+20}{(2x-3)^2}$

$= \frac{36x^2-108x+11}{(2x-3)^2}$

slope $= f'(-1) = \frac{36+108+11}{25} = \frac{155}{25} = \frac{31}{5}$

$\rightarrow y = \frac{31}{5}x + \frac{26}{5}$ #

1. (c) $[y^2(x^2+y^2)]' = (2x^2)'$

$2y\left(\frac{dy}{dx}\right)(x^2+y^2) + y^2(2x+2y\left(\frac{dy}{dx}\right)) = 4x$

$2y(x^2+y^2)\left(\frac{dy}{dx}\right) + 2xy^2 + 2y^3\left(\frac{dy}{dx}\right) = 4x$

$\left(\frac{dy}{dx}\right) = \frac{4x-2xy^2}{2x^2y+4y^3}$

slope $= \frac{4\times1-2\times1\times1^2}{2\times1^2\times1+4\times1^3} = \frac{2}{6} = \frac{1}{3}$

$\rightarrow y = \frac{1}{3}x + \frac{2}{3}$ #

2. (a) $y' = [\cot(8x^2+3)]'$

$\frac{dy}{dx} = -\csc^2(8x^2+3)\cdot(16x)$

$= -16x\csc^2(8x^2+3)$ #

2. (b) $f'(x) = 3(x^3-2x)^2\cdot(3x-2)$

$f''(x) = 6(x^3-2x)\cdot(3x-2)\cdot(3x-2) + 3(x^3-2x)^2\cdot 3$

$= 6(x^3-2x)(3x-2)^2 + 9(x^3-2x)(x^3-2x)$

$- 3(x^3-2x)[2(3x-2)^2 + 3(x^3-2x)]$

$= 3(x^3-2x)(3x^3+18x^2-30x+8)$

$\rightarrow f''(1) = 3(-1)(3+18-30+8) = 3$ #

3. $f'(x) = \frac{\sqrt{2x-1} - x\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{2x-1}}\cdot 2}{2x-1}$

$= \frac{\frac{2x-1-x}{\sqrt{2x-1}}}{2x-1}$

$= \frac{x-1}{(2x-1)\sqrt{2x-1}}$

set $f'(x) = m = 0 \rightarrow f'(1) = 0$

$\rightarrow (1,1)$ #

4. (a) $g'(x) = \sqrt{x+3} + x\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x+3}}$

$= \sqrt{x+3} + \frac{x}{2\sqrt{x+3}}$

$= \frac{2x+6+x}{2\sqrt{x+3}}$

$= \frac{3x+6}{2\sqrt{x+3}} = \frac{3(x+2)}{2\sqrt{x+3}} = \frac{3}{2}(x+2)(x+3)^{-\frac{1}{2}}$

$g''(x) = \frac{3}{2}\cdot1\cdot(x+3)^{-\frac{1}{2}} + \frac{3}{2}\cdot(x+2)\cdot(-\frac{1}{2})(x+3)^{-\frac{3}{2}}$

$\rightarrow g'(-2) = 0,\ g''(-2) = \frac{3}{2} > 0$

$\therefore$ relative min : $(-2,-2)$ #

no inflection point #

4. (b) $f'(x) = \frac{-4(2x)}{(x^2+1)^2} = \frac{-8x}{(x^2+1)^2}$

$f''(x) = \frac{-8(x^2+1)^2 + 8x\cdot2(x^2+1)\cdot(2x)}{(x^2+1)^4}$

$= \frac{-8(x^2+1)^2 + 32x^2(x^2+1)}{(x^2+1)^4}$

$\rightarrow f'(0) = 0,\ f''(0) = \frac{-8}{1} = -8 < 0$

$\therefore$ relative min : $(0,4)$ #

set $f''(x) = 0$

$\rightarrow (-8x^2-8)(x^2+1) + 32x^2(x^2+1) = 0$

$(24x^2-8)(x^2+1) = 0$

$(3x^2-1)(x^2+1) = 0$

$x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$\therefore$ inflection point : $(\frac{\sqrt{3}}{3}, 3),\ (-\frac{\sqrt{3}}{3}, 3)$ #

5. (a) $f''(x) = -2x + 2$

→ $f'(x)$ is increasing on $(-\infty, 1)$ #

$f'(x)$ is decreasing on $(1, \infty)$ #

(b) $f''(x) = -2x + 2$

$f$ is concave upward on $(-\infty, 1)$, concave downward on $(1, \infty)$ #

(c) $f'(1) = 0$, $f''(1) = 0$

∴ no relative extrema #

inflection point : $x = 1$ #

6. $f'(x) = \frac{4}{5} \cdot \frac{1}{2} \cdot (3-x)^{-\frac{1}{2}} \cdot (-1)$

$= \frac{-4}{6\sqrt{3-x}} = \frac{-2}{3\sqrt{3-x}}$

| | $f(0) = \frac{4\sqrt{3}}{5}$ | | $f(3) = 0$ |
|---|---|---|---|
| $f(x)$ | | ↘ | |
| $f'(x)$ | $x=0$ | − | $x=3$ |

∴ Max : $(0, \frac{4\sqrt{3}}{3})$, min : $(3, 0)$ #

榮譽第一

國立東華大學
應用數學系 _____學年度第___學期　考試科目：商用微積分　□期中□期末考試試卷

學號4111320001　姓名陳若涵　系所企管　班別一　任課教師：_____　共___張

1、a、

$f'(t) = 2t(t+2)^{\frac{1}{2}} + (t^2-9)\cdot\frac{1}{2}(t+2)^{-\frac{1}{2}}$

$= (t+2)^{-\frac{1}{2}}\left(2t(t+2) + \frac{1}{2}(t^2-9)\right)$

$= (t+2)^{-\frac{1}{2}}\left(2t^2+4t+\frac{1}{2}t^2-\frac{9}{2}\right)$,

$\Rightarrow f'(-1) = 2-4+\frac{1}{2}-\frac{9}{2} = -6$

$y+8 = -6(x+1) \Rightarrow y = -6x-14$ #

b、

$f'(x) = \frac{(36x+3)(2x-3)-(18x^2+3x-10)\cdot 2}{(2x-3)^2}$

$\Rightarrow f'(-1) = \frac{155}{25} = \frac{31}{5}$

$31x-5y = c$ at $(-1,-1)$

$\Rightarrow 31x-5y = -26$ #

c、

$2xy^2 + x^2 2y\frac{dy}{dx} + 4y^2\frac{dy}{dx} = 4x$

$\frac{dy}{dx} = \frac{4x-2xy^2}{2x^2y+4y^3}$ at $(1,1) \rightarrow \frac{dy}{dx} = \frac{1}{3}$

$x-3y = c$ at $(1,1)$

$\Rightarrow x-3y = -2$ #

2、a、

$y = \frac{1}{\tan(8x^2+3)}$

$\frac{dy}{dx} = \frac{-\sec^2(8x^2+3)\cdot 16x}{\tan^2(8x^2+3)}$

$= \frac{-16x}{\sin^2(8x^2+3)}$ #

b、

$f'(x) = 3(x^3-2x)^2(3x^2-2)$

$f''(x) = 6(x^3-2x)(3x^2-2)(3x^2-2) + 3(x^3-2x)^2\cdot 6x$

$f''(1) = -6+18 = 12$ #

3、

$f'(x) = \frac{(2x-1)^{\frac{1}{2}} - x\cdot\frac{1}{2}(2x-1)^{-\frac{1}{2}}\cdot 2}{2x-1}$

$= \frac{(2x-1)^{\frac{1}{2}} - x(2x-1)^{-\frac{1}{2}}}{(2x-1)}$

$= \frac{(2x-1)^{-\frac{1}{2}}(2x-1-x)}{(2x-1)}$

$= \frac{(2x-1)^{-\frac{1}{2}}(x-1)}{(2x-1)} = 0$, $x=1$

$f(1) = 1 \Rightarrow y = 1$ #

4、a、

$g'(x) = (x+3)^{\frac{1}{2}} + x\cdot\frac{1}{2}(x+3)^{-\frac{1}{2}}$

$= (x+3)^{-\frac{1}{2}}\left(x+3+\frac{1}{2}x\right)$

$= (x+3)^{-\frac{1}{2}}\cdot\frac{3}{2}(x+2) = 0$, $x=-2$

$g''(x) = -\frac{1}{2}(x+3)^{-\frac{3}{2}}\cdot\frac{3}{2}(x+2) + (x+3)^{-\frac{1}{2}}\cdot\frac{3}{2}$

$= (x+3)^{-\frac{3}{2}}\left(-\frac{1}{2}\cdot\frac{3}{2}(x+2) + (x+3)\cdot\frac{3}{2}\right)$

$= 0$, $x=-4$, $g(-4) \rightarrow$ undefined

$g''(-2) > 0$

$g(-2) = -2$ $(-2,-2)$ rel. min, no rel. max. 且 there are no inflection points of g #

b、

$f'(x) = \frac{-4(2x)}{(x^2+1)^2} = \frac{-8x}{(x^2+1)^2} = 0$, $x=0$

$f''(x) = \frac{-8(x^2+1)^2 + 16x(x^2+1)\cdot 2x}{(x^2+1)^4}$

$= \frac{8(x^2+1)(3x^2-1)}{(x^2+1)^4} = 0$, $x = \pm\frac{\sqrt{3}}{3}$

$f(\pm\frac{\sqrt{3}}{3}) = 3 \Rightarrow$ points of inflection

$(\frac{\sqrt{3}}{3}, 3)$ $(-\frac{\sqrt{3}}{3}, 3)$ #

$f(0) = 4 \rightarrow (0,4)$ rel. max. #

no rel. min. # $f''(0) > 0$

10

5、

$f''(x) = -2x+2 = -2(x-1) = 0$ $\quad x=1$ $\quad \frac{+\ |\ -}{CU\ ,\ CD}$

a、 increasing on $(-\infty, 1)$

decreasing on $(1, \infty)$ #

b、 concave upward on $(-\infty, 1)$

concave downward on $(1, \infty)$ #

c、 $f'(x) = -(x-1)^2 = 0$ , $x=1$ #

$f(x) = -\frac{1}{3}x^3 + x^2 - x$

$f(1) = -\frac{1}{3} + 1 - 1 = -\frac{1}{3}$

inflection point $(1, -\frac{1}{3})$ #

02

b、

$f'(x) = \frac{4}{3} \cdot \frac{1}{2}(3-x)^{-\frac{1}{2}}$

$= -\frac{2}{3}(3-x)^{-\frac{1}{2}} = 0 \rightarrow$ undefined

abs. extrema $\infty$ #