CalculusBM
Instructor: Yu-Ling Tseng

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Quiz 1
20221012

94

* **Note:** No points will be given if no arguments are provided for an answer. *Good Luck!* ~~ *Yuling* ◡̈

10

1. (10 points) Find the domain and range of the function

$$f(x) = \frac{x-2}{x+4}$$

$x+4 \neq 0$, $x \neq -4$

$y = \frac{x-2}{x+4}$

domain of the function : $x \neq -4$ #

$y(x+4) = x-2$

$xy+4y = x-2$

$xy - x = -4y-2$

$x(y-1) = -4y-2$

$x = \frac{-4y-2}{y-1}$

$y-1 \neq 0$

$y \neq 1$

range of the function : $y \neq 1$ #

10

2. (10 points) Determine whether the function $f(x) = |x+3|$ is one-to-one. If it is, find its inverse function.

$f(x) = |x+3|$ is not one-to-one

(ex. $x = 1$ $f(x) = 4$
$x = -7$ $f(x) = 4$)

10

3. (10 points) Find the inverse function of f, where $f(x) = \sqrt{9-x^2}$, $0 \leq x \leq 3$.

$y = \sqrt{9-x^2}$

$x = \sqrt{9-y^2}$

$x = (9-y^2)^{\frac{1}{2}}$

$x^2 = 9-y^2$

$y^2 = 9-x^2$

$y = \sqrt{9-x^2}$

$f^{-1}(x) = \sqrt{9-x^2}$ $0 \leq x \leq 3$ #

10

4. (10 points) Describe the interval(s) on which the function is continuous. If there are any discontinuities, determine whether they are removable.

$$f(x) = \begin{cases} x^2+1 & x < 0 \\ x-1 & x \geq 0 \end{cases}$$

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x^2+1) = 1$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x-1) = -1$

$\therefore \lim_{x \to 0} f(x)$ is not exist

→ nonremovable discontinuity

10

5. (10 points) Find the constant a such that the function $f(x)$ is continuous on the entire real number line, where

$$f(x) = \begin{cases} x^3 & x \leq 2 \\ ax^2 & x > 2 \end{cases}$$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^3 = 8$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} ax^2 = 4a$

$8 = 4a$, $a = 2$ #

6. (50 points) Find the indicated limit or show it does not exist. If the limiting value is infinite, indicate whether it is ∞ or $-\infty$.

$$(a)\ \lim_{x\to 2}\frac{x^2+3x-10}{x^2-4},\quad (b)\ \lim_{\Delta t\to 0}\frac{(t+\Delta t)^2-5(t+\Delta t)-(t^2-5t)}{\Delta t},$$

$$(c)\ \lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x},\quad (d)\ \lim_{x\to -3^-}\frac{|x+3|}{x+3},\quad (e)\ \lim_{x\to -2^+}\frac{3}{x^2-4}$$

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a. $\lim_{x\to 2}\frac{x^2+3x-10}{x^2-4}$

$= \lim_{x\to 2}\frac{(x+5)(x-2)}{(x+2)\cancel{(x-2)}}$

$= \frac{7}{4}$ #

b. $\lim \frac{(t+\Delta t)^2-5(t+\Delta t)-(t^2-5t)}{\Delta t}$

$= \frac{\cancel{t^2}+2t\Delta t+\Delta t^2-\cancel{5t}-5\Delta t-\cancel{t^2}+\cancel{5t}}{\Delta t}$

+7

$= 2t+\cancel{\Delta t}-5$

$\lim_{\Delta t\to 0}(2t+\Delta t-5) = 2t-5$ #

c. $\lim_{x\to 0}\frac{(\sqrt{x+5}-\sqrt{5})(\sqrt{x+5}+\sqrt{5})}{x(\sqrt{x+5}+\sqrt{5})}$

$= \lim_{x\to 0}\frac{1}{\sqrt{x+5}+\sqrt{5}}$

$= \frac{1}{2\sqrt{5}}$ #

d. $\lim_{x\to -3^-}\frac{|x+3| \to \oplus}{x+3 \to \ominus}$

$= -1$ #

e. $\lim_{x\to -2^+}\frac{3}{x^2-4}$

the limit is not exist, because it unbounded as x approach -2 from the right. #

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Quiz 1
20221012

100

* **Note:** No points will be given if no arguments are provided for an answer.
Good Luck! ~~ *Yuling* :)

10

1. (10 points) Find the domain and range of the function

$$f(x) = \frac{x-2}{x+4}$$

$f(-4)$ is not defined
so Domain of function $(-\infty,-4)(-4,\infty)$

$y = \frac{x-2}{x+4}$

$y(x+4) = x-2$

$yx + 4y = x-2$

$yx - x = -4y-2$

$x(y-1) = -4y-2$

$x = \frac{-4y-2}{y-1}$

$f^{-1}(x) = \frac{-4x-2}{x-1}$

Domain: $(-\infty,-4),(-4,\infty)$

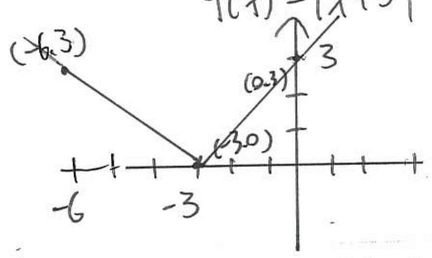
A: Range: $(-\infty,1),(1,\infty)$

f(x)的值域=$f^{-1}(x)$的定義域
∵ domain of the function $f^{-1}(x)$: $(-\infty,1),(1,\infty)$
∴ range of the function $f(x)$: $(-\infty,1),(1,\infty)$

10

2. (10 points) Determine whether the function $f(x) = |x+3|$ is one-to-one. If it is, find its inverse function.

$f(x) = |x+3|$



$x=0$ $y=3$
$x=-6$ $y=3$

A: the function $f(x)=|x+3|$ is not one-to-one

10

3. (10 points) Find the inverse function of f, where $f(x) = \sqrt{9-x^2}$, $0 \le x \le 3$.

$y = \sqrt{9-x^2}$

$y^2 = 9-x^2$

$x^2 = 9-y^2$

$9-x^2 \geq 0$

$x = \sqrt{9-y^2}$

$f^{-1}(x) = \sqrt{9-x^2}$ $0 \le x \le 3$

A: $f^{-1}(x) = \sqrt{9-x^2}$, $0 \le x \le 3$

10

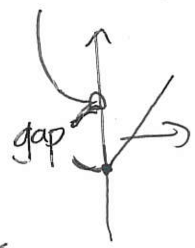
4. (10 points) Describe the interval(s) on which the function is continuous. If there are any discontinuities, determine whether they are removable.

$$f(x) = \begin{cases} x^2+1 & x < 0 \\ x-1 & x \geq 0 \end{cases}$$

$\lim_{x\to 0^+} f(x) = -1$

$\lim_{x\to 0^-} f(x) = 1$

$\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$



A: Continuous on $(-\infty,0)$ and $(0,\infty)$; there is a nonremovable discontinuity at $x=0$

10

5. (10 points) Find the constant a such that the function $f(x)$ is continuous on the entire real number line, where

$$f(x) = \begin{cases} x^3 & x \le 2 \\ ax^2 & x > 2 \end{cases}$$

$\lim_{x\to 2^+} f(x) = 4a$

$\lim_{x\to 2^-} f(x) = 8$

$\lim_{x\to 2} f(x) = 8$

$4a = 8$

$a = 2$

A: $a=2$

6. (50 points) Find the indicated limit or show it does not exist. If the limiting value is infinite, indicate whether it is ∞ or $-\infty$.

$$(a)\ \lim_{x\to2}\frac{x^2+3x-10}{x^2-4},\quad (b)\ \lim_{\Delta t\to0}\frac{(t+\Delta t)^2-5(t+\Delta t)-(t^2-5t)}{\Delta t},$$

$$(c)\ \lim_{x\to0}\frac{\sqrt{x+5}-\sqrt{5}}{x},\quad (d)\ \lim_{x\to-3^-}\frac{|x+3|}{x+3},\quad (e)\ \lim_{x\to-2^+}\frac{3}{x^2-4}$$

50

(a.) $\lim_{x\to2}\frac{(x+5)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$

$= \lim_{x\to2}\frac{x+5}{x+2} = \frac{7}{4}$ #

(b.) $\lim_{\Delta t\to0}\frac{(t+\Delta t)^2-5(t+\Delta t)-(t^2-5t)}{\Delta t}$

$= \lim_{\Delta t\to0}\frac{\cancel{t^2}+2\cdot t\Delta t+\Delta t^2\cancel{-5t}-5\Delta t\cancel{-t^2}\cancel{+5t}}{\Delta t}$

$= \lim_{\Delta t\to0}\frac{2\cdot t\Delta t+\Delta t^2-5\Delta t}{\Delta t} = \lim_{\Delta t\to0} 2t+\Delta t-5$

$= 2t-5,\ \Delta t\neq0$ #

(c.) $\lim_{x\to0}\frac{\sqrt{x+5}-\sqrt{5}\,(\sqrt{x+5}+\sqrt{5})}{x\,(\sqrt{x+5}+\sqrt{5})}$

$= \lim_{x\to0}\frac{\cancel{x}}{\cancel{x}(\sqrt{x+5}+\sqrt{5})} = \lim_{x\to0}\frac{1}{\sqrt{x+5}+\sqrt{5}}$

$= \frac{1}{2\sqrt{5}}$ #

(d.) $\lim_{x\to-3^-}\frac{|x+3|}{x+3} = \frac{-x-3}{x+3} = -1$ #

-3

(e.) $\lim_{x\to2^+}\frac{3}{x^2-4} = \lim_{x\to2^+}\frac{3}{0.00000\ldots}$

-2^+ -2^+

2

$= -\infty$ #