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榮譽第一

國立東華大學 應用數學系 ______學年度第____學期　考試科目：________________ □期中□期末考試試卷

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(a) $\int 5x\sqrt[3]{1-x^2}\,dx$ $\quad u=1-x^2 \quad du=-2x\,dx$

$= -\frac{5}{2}\int u^{\frac{1}{3}}\,du$

$= -\frac{15}{8}(1-x^2)^{\frac{4}{3}} + C$ #

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(b) $\int \frac{5}{e^{-5x}+7}\,dx$

$= \int \frac{5e^{5x}}{7e^{5x}+1}\,dx$ $\quad u=7e^{5x}+1 \quad du=35e^{5x}dx$

$= \frac{1}{7}\int \frac{1}{u}\,du$

$= \frac{1}{7}\ln|7e^{5x}+1| + C$ #

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(c) $\int \frac{x}{\sqrt{x-1}}\,dx$ $\quad u=x \quad du=dx$

$= 2x(x-1)^{\frac{1}{2}} - 2\int (x-1)^{\frac{1}{2}}dx$ $\quad dv=(x-1)^{-\frac{1}{2}}dx \quad v=2(x-1)^{\frac{1}{2}}$

$= 2x(x-1)^{\frac{1}{2}} - \frac{4}{3}(x-1)^{\frac{3}{2}} + C$ #

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(d) $\int \frac{e^{-x}}{1-e^{-x}}\,dx$ $\quad u=1-e^{-x} \quad du=e^{-x}dx$

$= \int \frac{1}{u}\,du$

$= \ln|1-e^{-x}| + C$ #

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(e) $\int t\ln(t+1)\,dt$ $\quad u=\ln(t+1) \quad du=\frac{1}{t+1}dt$

$= \frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\int \frac{t^2}{t+1}dt$ $\quad dv=t\,dt \quad v=\frac{1}{2}t^2$

$= \frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\int (t-1) + \frac{1}{t+1}\,dt$

$= \frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\left[\frac{1}{2}t^2 - t + \ln|t+1|\right] + C$

$= \frac{1}{2}t^2\ln(t+1) - \frac{1}{4}t^2 + \frac{1}{2}t - \frac{1}{2}\ln|t+1| + C$ #

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(f) $\int x^2(\ln x)^3\,dx$ $\quad u=(\ln x)^3 \quad du=3\ln x^2\cdot\frac{1}{x}dx$

$= \frac{1}{3}x^3(\ln x)^3 - \int x^2(\ln x)^2\,dx$ $\quad dv=x^2dx \quad v=\frac{1}{3}x^3$

$= \frac{1}{3}x^3(\ln x)^3 - \left[\frac{1}{3}x^3(\ln x)^2 - \frac{2}{3}\int x^2\ln x\,dx\right]$ $\quad u'=(\ln x)^2 \quad du=2\ln x\cdot\frac{1}{x}dx$

$= \frac{1}{3}x^3(\ln x)^3 - \frac{1}{3}x^3(\ln x)^2 + \frac{2}{3}\left[\frac{1}{3}x^3(\ln x) - \frac{1}{9}x^3\right] + C$ $\quad u=\ln x \quad du=\frac{1}{x}dx$

$= \frac{1}{3}x^3(\ln x)^3 - \frac{1}{3}x^3(\ln x)^2 + \frac{2}{9}x^3(\ln x) - \frac{2}{27}x^3 + C$ #

2. $f'(x) = -3x^2(2-x^3)^4$

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$\Rightarrow \int -3x^2(2-x^3)^4\,dx$ $\quad u=2-x^3 \quad du=-3x^2dx$

$= \int u^4\,du$

$= \frac{1}{5}(2-x^3)^5 + C \quad \rightarrow$ 代 $(0,7)$

$\Rightarrow f(x) = \frac{1}{5}(2-x^3)^5 + \frac{3}{5}$ #

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3. $f''(x) = 2 \rightarrow f'(x) = 2x + C \Rightarrow C = 1$

$\Rightarrow f'(x) = 2x+1$

$\Rightarrow \int (2x+1)\,dx = x^2 + x + C = f(x)$

$\Rightarrow f(2) = 10 \Rightarrow C = 4 \Rightarrow f(x) = x^2 + x + 4$ #

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4. (a) $\int (\sin x + \cos x)^2\,dx$

$= \int (1 + 2\sin x\cos x)\,dx$

$= \int (1 + \sin 2x)\,dx$

$= x - \frac{1}{2}\cos 2x + C$ #

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(b) $\int 6x\sec^2(x)\,dx$ $\quad u=6x \quad du=6$

$= 6x\tan x - \int 6\tan x\,du$ $\quad dv=\sec^2 x\,dx \quad v=\tan x$

$= 6x\tan x + 6\ln|\cos x| + C$ #

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(10)

1.(a). $\int 5x\sqrt[3]{1-x^2}\,dx$ $\quad u=x^2,\ du=2x\,dx,\ \frac{1}{2}du=x\,dx$

$=\int 5x(1-x^2)^{\frac{1}{3}}\,dx$

$=\frac{1}{2}5\int(1-u)^{\frac{1}{3}}\,du$

$=\frac{5}{2}\int(1-u)^{\frac{1}{3}}-d(1-u)$

$=-\frac{5}{2}\int(1-u)^{\frac{1}{3}}\,d(1-u)$

$=-\frac{5}{2}\,\frac{3}{4}(1-u)^{\frac{4}{3}}+C$

$=-\frac{15}{8}(1-x^2)^{\frac{4}{3}}+C$ #

(10)

(b) $\int\frac{5}{e^{(-5x)}+7}\,dx$ $\quad u=-5x,\ du=-5dx,\ -du=5dx$

$=-\int\frac{1}{e^{u}+7}\,du$

$=-\int\frac{e^{-u}}{1+7e^{-u}}\,du$

$=-\int\frac{1}{7e^{-u}+1}\,d-e^{-u}$

$=\frac{1}{7}\int\frac{1}{7e^{-u}+1}\,d7e^{-u}$

$=\frac{1}{7}\int\frac{1}{7e^{-u}+1}\,d(7e^{-u}+1)$

$=\frac{1}{7}\ln|7e^{-u}+1|+C$

$=\frac{1}{7}\ln|7e^{5x}+1|+C$ #

(10)

(c). $\int\frac{x}{\sqrt{x-1}}\,dx$

$=\int\frac{x-1+1}{(x-1)^{\frac{1}{2}}}\,dx$

$=\int\frac{(x-1)}{(x-1)^{\frac{1}{2}}}+\frac{1}{(x-1)^{\frac{1}{2}}}\,dx$

$=\int(x-1)^{\frac{1}{2}}+(x-1)^{-\frac{1}{2}}\,dx$

$=\int(x-1)^{\frac{1}{2}}\,d(x-1)+\int(x-1)^{-\frac{1}{2}}\,d(x-1)$

$=\frac{2}{3}(x-1)^{\frac{3}{2}}+2(x-1)^{\frac{1}{2}}+C$ #

(10)

(d). $\int\frac{e^{-x}}{1-e^{-x}}\,dx$

$=\int\frac{1}{1-e^{-x}}\,d-e^{-x}$

$=\int\frac{1}{(-e^{-x}+1)}\,d(-e^{-x}+1)$

$=\ln|-e^{-x}+1|+C$ #

(10)

(e). $\int t\ln(t+1)\,dt$ $\quad u=t+1,\ du=dt,\ t=u-1$

$=\int(u-1)\ln(u)\,du$

$=\int u\cdot\ln(u)-\ln(u)\,du$

$=\underbrace{\int u\cdot\ln(u)\,du}_{A}-\underbrace{\int\ln(u)\,du}_{B}$

$A=\int u\cdot\ln(u)\,du=\int\ln(u)\,d\frac{1}{2}u^2$

$=\ln(u)\cdot\frac{1}{2}u^2-\int\frac{1}{2}u^{\cancel{2}}\cdot\frac{1}{\cancel{u}}\,du$

$=\frac{1}{2}\ln(u)u^2-\frac{1}{2}\int u\,du$

$=\frac{1}{2}\ln(u)u^2-\frac{1}{2}\cdot\frac{1}{2}u^2+C$

$=[\frac{1}{2}\ln(u)\cdot u^2-\frac{1}{4}u^2+C]$

$B=\int\ln(u)\,du=\ln(u)\cdot u-\int u\cdot\frac{1}{u}\,du$

$=[\ln(u)\cdot u-u+C]$

$=A-B=\frac{1}{2}\ln(t+1)\cdot(t+1)^2-\frac{1}{4}(t+1)^2$

$-[\ln(t+1)\cdot(t+1)-(t+1)]+C$

$=\frac{1}{2}(t+1)^2\ln(t+1)-\frac{1}{4}(t+1)^2-(t+1)\ln(t+1)+(t+1)+C$ #

(f). $\int x^2(\ln x)^3\,dx$

$=\int(\ln x)^3\,d\frac{1}{3}x^3$

$=(\ln x)^3\cdot\frac{1}{3}x^3-\int\frac{1}{3}x^{\cancel{3}2}\cancel{3}(\ln x)^2\cdot\frac{1}{\cancel{x}}\,dx$

$=(\ln x)^3\cdot\frac{1}{3}x^3-\boxed{\int x^2(\ln x)^2\,dx}\ A$

(10)

$A=\int(\ln x)^2x^2\,dx=\int(\ln x)^2\,d\frac{1}{3}x^3$

$=(\ln x)^2\cdot\frac{1}{3}x^3-\int\frac{1}{3}x^{\cancel{3}2}\cdot2(\ln x)\cdot\frac{1}{\cancel{x}}\,dx$

$=(\ln x)^2\cdot\frac{1}{3}x^3-\frac{2}{3}\int x^2(\ln x)\,dx$

$=(\ln x)^2\cdot\frac{1}{3}x^3-\frac{2}{3}[\int(\ln x)\cdot d\frac{1}{3}x^3]$

$=(\ln x)^2\cdot\frac{1}{3}x^3-\frac{2}{3}[\ln(x)\cdot\frac{1}{3}x^3-\int\frac{1}{3}x^{\cancel{3}2}\cdot\frac{1}{\cancel{x}}\,dx]$

$=[(\ln x)^2\cdot\frac{1}{3}x^3-\frac{2}{3}[\ln(x)\cdot\frac{1}{3}x^3-\frac{1}{3}\cdot\frac{1}{3}x^3]]\ A$

$=(\ln x)^3\frac{1}{3}x^3-(\ln x)^2\cdot\frac{1}{3}x^3+\frac{2}{3}(\ln(x)\cdot\frac{1}{3}x^3-\frac{1}{9}x^3)$

$=(\ln x)^3\frac{1}{3}x^3-(\ln x)^2\cdot\frac{1}{3}x^3+\frac{2}{3}\ln(x)\frac{1}{3}x^3-\frac{2}{27}x^3+C$ #

2、$\int f'(x)\,dx = f(x)$

$f(x)=\int -3x^2(2-x^3)^4\,dx \qquad u=x^3 \quad -du=-3x^2dx.$

$=-\int(2-u)^4\,du.$

$=-\int(2-u)^4 - d(2-u)$

$=\int(2-u)^4\,d(2-u)$

$=\frac{1}{5}(2-u)^5+C$

$=\frac{1}{5}(2-x^3)^5+C$

套入 $x=0 \quad y=7$

$7=\frac{1}{5}(2-0)^5+C$

$7-\frac{32}{5}=C.$

$C=\frac{3}{5}.$

故 $f(x)=\frac{1}{5}(2-x^3)^5+\frac{3}{5}$ #

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3、$\int f''(x)\,dx=f'(x)$

$f(x)=\int 2\,dx$

$f(x)=2x+C$

$f(2)=4+C=5 \Rightarrow C=1.$

$\therefore f'(x)=2x+1.$

$\int f'(x)\,dx = f(x).$

$f(x)=\int(2x+1)\,dx$

$=2\int x\,dx+\int dx$

$=2\cdot\frac{1}{2}x^2+x+C$

$f(x)=x^2+x+C.$

$f(2)=4+2+C=10$

$C=4$

故 $f(x)=x^2+x+4$ #

10

4.(a). $\int(\sin(x)+\cos(x))^2\,dx$

$=\int 1+2\sin(x)\cos(x)\,dx$

$=\int 1\,dx+2\int\sin(x)\cos(x)\,dx$

$=x+2\int\sin(x)\,d\sin(x)$

$=x+2\cdot\frac{1}{2}\sin^2(x)+C.$

$=x+\sin^2(x)+C$ #

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(b). $\int 6x\sec^2(x)\,dx$

$=\int 6x\,d\tan(x).$

$=6x\cdot\tan(x)-\int\tan(x)\cdot 6\,dx.$

$=6x\cdot\tan(x)-6\int\tan(x)\,dx$

$=6x\cdot\tan(x)-6\int\frac{\sin(x)}{\cos(x)}\,dx.$

$=6x\cdot\tan(x)-6\int\frac{1}{\cos(x)}\,d-\cos(x)$

$=6x\cdot\tan(x)+6\int\frac{1}{\cos(x)}\,d\cos(x)$

$=6x\cdot\tan(x)+6\ln|\cos(x)|+C$ #

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