

榮譽第一

國立東華大學
應用數學系

109 學年度第 1 學期

考試科目：微積分

期中 期末 考試試卷

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10	<p>1. $f(x) = \frac{x-2}{x+4}$</p> <p>Domain: 分母不能為 0, $x+4 \neq 0, x \neq -4$ $x \in (-\infty, -4) \cup (-4, \infty)$</p> <p>Range: $\frac{x+4-b}{x+4} = 1 - \frac{b}{x+4} \neq 0, y \neq 1$ $y \in (-\infty, 1) \cup (1, \infty)$</p>	<p>10 5. $f(x) = \frac{1}{x-2} [1, 4]$</p> <p>當 $x \in [1, 2) \cup (2, 4]$ conti</p> <p>當 $x = 2$ not conti</p> <p>nonremovable discontinuity at $x=2 \Rightarrow$ 不連續</p>	<p>$\lim_{x \rightarrow 2^+} \frac{1}{x-2} \rightarrow +\infty$</p> <p>$\lim_{x \rightarrow 2^-} \frac{1}{x-2} \rightarrow -\infty$</p> <p>$\lim_{x \rightarrow 2} \frac{1}{x-2}$ 不存在</p>
10	<p>2. $f(x) = x+3$</p> <p>it is not one-to-one</p>	<p>10 6. $f(x) = \begin{cases} x^3 & x \leq 2 \\ ax^2 & x > 2 \end{cases}$</p> <p>if $x=2, 2^3=8$ $4a=8$</p>	<p>$\lim_{x \rightarrow 2} f(x) = 8$</p> <p>$\Rightarrow$ 極限存在 \Rightarrow 連續</p>
10	<p>3. $f(x) = \sqrt{9-x^2}, 0 \leq x \leq 3$</p>	<p>because it is not 遞增 or 遞減 函數</p>	<p>$a=2$</p>
10	<p>$y = \sqrt{9-x^2} \Rightarrow y^2 = 9-x^2 \Rightarrow x^2 = 9-y^2$</p> <p>$\Rightarrow x = \sqrt{9-y^2} = g(y)$</p> <p>$\Rightarrow f^{-1}(x) = \sqrt{9-x^2}, 0 \leq x \leq 3$</p>		
10	<p>4-a $\lim_{x \rightarrow 3} \frac{x^2-9}{x+3}$</p> <p>$\frac{(x+3)(x-3)}{x+3} = x-3$</p> <p>$\lim_{x \rightarrow 3} x-3 = 0$</p>		
10	<p>4-b $\lim_{\Delta t \rightarrow 0} \frac{(t+\Delta t)^2 - 5(t+\Delta t) - (t^2 - 5t)}{\Delta t}$</p> <p>$\frac{t^2 + 2t\Delta t + \Delta t^2 - 5t - 5\Delta t - t^2 + 5t}{\Delta t}$</p> <p>$= \frac{2t\Delta t + \Delta t^2 - 5\Delta t}{\Delta t} = 2t + \Delta t - 5 \Rightarrow 2t - 5$</p> <p>$\lim_{\Delta t \rightarrow 0} (2t + \Delta t - 5) = 2t - 5$</p>		
10	<p>4-c $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x-4}$</p> <p>$\frac{(\sqrt{x+5}-3)(\sqrt{x+5}+3)}{x-4(\sqrt{x+5}+3)} = \frac{x+5-9}{x-4(\sqrt{x+5}+3)} = \frac{1}{\sqrt{x+5}+3}$</p> <p>$\lim_{x \rightarrow 4} \frac{1}{\sqrt{x+5}+3} = \frac{1}{\sqrt{4+5}+3} = \frac{1}{6}$</p>		
10	<p>4-d $\lim_{x \rightarrow 0} \frac{\sqrt{x+5} - \sqrt{5}}{x}$</p> <p>$\frac{(\sqrt{x+5}-\sqrt{5})(\sqrt{x+5}+\sqrt{5})}{x(\sqrt{x+5}+\sqrt{5})} = \frac{x+5-5}{x(\sqrt{x+5}+\sqrt{5})} = \frac{1}{\sqrt{x+5}+\sqrt{5}}$</p> <p>$\lim_{x \rightarrow 0} \frac{1}{\sqrt{x+5}+\sqrt{5}} = \frac{1}{2\sqrt{5}}$</p>		
10	<p>4-e $\lim_{x \rightarrow -2} \frac{1}{x+2}$</p> <p>stop here</p> <p>$\lim_{x \rightarrow -2^-} \frac{1}{x+2} \rightarrow -\infty$</p> <p>$\lim_{x \rightarrow -2^+} \frac{1}{x+2} \rightarrow +\infty$</p>	<p>It's only use to show $\lim_{x \rightarrow a} f(x)$ exists or not.</p> <p>The question ask you to find $\lim_{x \rightarrow -2} \frac{1}{x+2}$, not $\lim_{x \rightarrow -2} \frac{1}{x+2}$</p> <p>左極限不存在</p>	<p>故 $\lim_{x \rightarrow -2} \frac{1}{x+2}$ D.N.E</p>

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10 1. domain: $\mathbb{R} \setminus \{-4\}$ $\leftarrow \because f(-4)$ D.N.E ~~$\frac{0}{0}$~~ $\neq 0$ (e) $\lim_{x \rightarrow -2} \frac{1}{x+2} = \frac{1}{0^-} = -\infty$ (D.N.E)

range: $\mathbb{R} \setminus \{1\}$ $\leftarrow \because x-2 \neq x+4$

10 2. $f(x) = |x+3|$ isn't one-to-one because it isn't the decrease or increase line

10 3. $f(x) = \sqrt{9-x^2}$
 $y = \sqrt{9-x^2}$
 $y^2 = 9-x^2$
 $x^2 = 9-y^2$
 $x = \sqrt{9-y^2} \Rightarrow x^{-1} = \sqrt{9-x^2}$
 \therefore the inverse function of f is $\sqrt{9-x^2}$, $0 \leq x \leq 3$

10 4. (a) $\lim_{x \rightarrow -3} \frac{x^2-9}{x+3} = \lim_{x \rightarrow -3} \frac{(x+3)(x-3)}{x+3} = \lim_{x \rightarrow -3} (x-3) = -3-3 = -6$

10 (b) $\lim_{\Delta t \rightarrow 0} \frac{(t+\Delta t)^2 - 5(t+\Delta t) - (t^2 - 5t)}{\Delta t}$
 $= \lim_{\Delta t \rightarrow 0} \frac{t^2 + 2t\Delta t + \Delta t^2 - 5t - 5\Delta t - t^2 + 5t}{\Delta t}$
 $= \lim_{\Delta t \rightarrow 0} \frac{2t\Delta t + \Delta t^2 - 5\Delta t}{\Delta t}$
 $= \lim_{\Delta t \rightarrow 0} (2t + \Delta t - 5)$
 $= 2t + 0 - 5 = 2t - 5$ ($\Delta t \neq 0$)

10 (c) $\lim_{x \rightarrow 4} \frac{\sqrt{x+5}-3}{x-4} \times \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$
 $= \lim_{x \rightarrow 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$
 $= \lim_{x \rightarrow 4} \frac{x-4}{(x-4)(\sqrt{x+5}+3)}$
 $= \frac{1}{\sqrt{9}+3}$
 $= \frac{1}{6}$

10 (d) $\lim_{x \rightarrow 0} \frac{\sqrt{x+5}-\sqrt{5}}{x}$
 $= \lim_{x \rightarrow 0} \frac{\sqrt{x+5}-\sqrt{5}}{x} \times \frac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}}$
 $= \lim_{x \rightarrow 0} \frac{x+5-x-5}{x(\sqrt{x+5}+\sqrt{5})} = \frac{1}{\sqrt{5}+\sqrt{5}} = \frac{1}{2\sqrt{5}} = \frac{\sqrt{5}}{10}$

10 5. $f(x) = \frac{1}{x-z}$
 $x \neq z \rightarrow f(z)$ 在 $[1, 4]$ 之間
 $\therefore f(z)$ D.N.E \because discontinuity, \because discontinuity \therefore Nonremovable
 \Rightarrow Nonremovable discontinuity at $x=z$

6. $\lim_{x \rightarrow z^+} f(x) = a \times z^2$
 $\lim_{x \rightarrow z^-} f(x) = z^3 = 8$
 $\lim_{x \rightarrow z} f(x) = z^3 = 8$
 $\therefore \lim_{x \rightarrow z^+} f(x) = \lim_{x \rightarrow z^-} f(x) = \lim_{x \rightarrow z} f(x)$
 $\therefore a \times z^2 = 8 \Rightarrow a = z$