Sol. to Quiz 4. Cal. BM.

#1. $\int_1^5 \frac{3}{x+2}\,dx = 3\ln|x+2|\Big|_1^5 = 3\ln 7 - 3\ln 3$

$\Rightarrow$ Average value of $f(x)=\frac{3}{x+2}$ on $[1,5]$ is $\frac{\int_1^5 f(x)dx}{5-1}$

$= \frac14(3\ln 7 - 3\ln 3) = \frac34\ln(\frac73)$

#2 (a) $\int_0^1 e^{2x}\sqrt{e^{2x}+1}\,dx$ 令 $u=e^{2x}+1$ $du=2e^{2x}dx$

$=\frac12\int_2^{e^2+1} u^{\frac12}du = \frac12\cdot\frac23 u^{\frac32}\Big|_2^{e^2+1} = \frac13(e^2+1)^{\frac32} - \frac13 2\sqrt2$

(b) $\int_2^8 |3x-9|\,dx$ $\quad 3x-9=0 \Leftrightarrow x=3$

$=\int_2^3 9-3x\,dx + \int_3^8 3x-9\,dx$

$= 9-3\frac{x^2}{2}\Big|_2^3 + 3\frac{x^2}{2}\Big|_3^8 - 9\times5$

$=9-3(\frac92-\frac42)+3(\frac{64}{2}-\frac92)-45=39$

(c) $\int_1^e x^5\ln x\,dx = \int_1^e \frac16 \ln x\, dx^6$

$=\frac16\ln x\cdot x^6\Big|_1^e - \int_1^e \frac16 x^{6}\cdot\frac1x\,dx$

$=\frac{e^6}{6} - \frac1{36}x^6\Big|_1^e = \frac{e^6}{6}-\frac{e^6}{36}+\frac1{36} = \frac5{36}e^6+\frac1{36}$

(d) $\int_1^2 x^2e^{2x}dx = \int_1^2 \frac12 x^2\, de^{2x}$

$=\frac12x^2e^{2x}\Big|_1^2 - \int_1^2 e^{2x}\,x\,dx = (\frac12x^2e^{2x} - \int\frac12 x\,de^{2x})\Big|_1^2$

$=(\frac12x^2e^{2x}-\frac x2e^{2x}+\int\frac12e^{2x}dx)\Big|_1^2 = (\frac12x^2e^{2x}-\frac x2e^{2x}+\frac14e^{2x})\Big|_1^2$

$=e^{2x}(\frac{x^2}{2}-\frac x2+\frac14)\Big|_1^2 = e^4(\frac42-\frac22+\frac14)-e^2(\frac12-\frac12+\frac14)$

$=\frac54e^4-\frac14e^2$

(e) $\int_0^8 x\sqrt{x+1}\,dx$ 令 $u=x+1$ $du=dx$, $x=u-1$

$=\int_1^9(u-1)u^{\frac12}du = \int_1^9 u^{\frac32}du - \int_1^9 u^{\frac12}du$

$=2u^{\frac32}(\frac15u-\frac13)\Big|_1^9 = 2\times9^{\frac32}(\frac95-\frac13)-(2(\frac15-\frac13)) = \frac{1192}{15}$

(f) $\int\frac{xe^{2x}}{(2x+1)^2}dx$ $\quad u=\frac{e^{2x}}{2x+1}$

$du=\frac{(2x+1)e^{2x}\cdot2-e^{2x}\cdot2}{(2x+1)^2}dx = \frac{4xe^{2x}}{(2x+1)^2}dx$

$=\int\frac14\,du$

$=\frac14u+C = \frac{e^{2x}}{4(2x+1)}+C$

(g) $\int\frac{x}{x^4-36}dx = \int\frac{x}{(x^2+6)(x^2-6)}dx$

$=\frac1{12}\int(\frac{x}{x^2-6}-\frac{x}{x^2+6})dx = \frac1{24}\int\frac{2x}{x^2-6}dx - \frac1{24}\int\frac{2x}{x^2+6}dx$

$=\frac1{24}\int\frac1{x^2-6}d(x^2-6)-\frac1{24}\int\frac1{x^2+6}d(x^2+6)$

$=\frac1{24}\ln\left|\frac{x^2-6}{x^2+6}\right|+C$

(h) $A=\int\sqrt{3+x^2}\,dx = \sqrt3\int\sqrt{1+(\frac{x}{\sqrt3})^2}\,dx$

令 $\frac{x}{\sqrt3}=\tan y \Rightarrow dx=\sqrt3\sec^2y\,dy$

$A=\sqrt3\int\sqrt{1+\tan^2y}\cdot\sqrt3\sec^2y\,dy$

$=3\int\sec^3y\,dy = 3\int\sec y\,d\tan y$

$=3\sec y\tan y - 3\int\tan y\cdot\sec y\tan y\,dy$

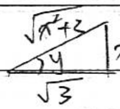
$=3\sec y\tan y-3\int\sec^3y\,dy+3\int\sec y\,dy$

$\Rightarrow 2A = 3\sec y\tan y+3\int\sec y\,dy$

$=3\sec y\tan y+3\int\sec y\left(\frac{\sec y+\tan y}{\sec y+\tan y}\right)dy$

$=3\sec y\tan y+3\int\frac{1}{\sec y+\tan y}\,d(\sec y+\tan y)$

$=3\sec y\tan y+3\ln(|\sec y+\tan y|)+C$



$\sec y=\frac{\sqrt{x^2+3}}{\sqrt3}$

$\Rightarrow\int\sqrt{3+x^2}\,dx=\frac12\left[3\times\frac{\sqrt{x^2+3}}{\sqrt3}\times\frac{x}{\sqrt3}+3\ln\left|\frac{\sqrt{x^2+3}}{\sqrt3}+\frac{x}{\sqrt3}\right|\right]+C$

$=\frac12x\sqrt{x^2+3}+\frac32\ln|\sqrt{x^2+3}+x|+C$

(i) $\int6x\sec^2x\,dx=\int6x\,d\tan x$

$=6x\tan x-\int6\tan x\,dx$

$=6x\tan x-6\int\frac{\sin x}{\cos x}dx$

$=6x\tan x-6\int\frac{1}{\cos x}\,d\cos x$

$=6x\tan x-6\ln|\cos x|+C$